Modeling the Temperature Dependence of Tertiary Creep Damage of a Ni-Based Alloy

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1 Introduction

Gas turbines generate electric power through a compression-combustion process that forces superheated highly pressurized exhaust gas through various components. High pressure in the compression area coupled with combustion chamber exit temperatures of up to 1350°C creates a situation where material design and selection play a major role in the long term reliability of gas turbine parts. Drives to increase gas turbine efficiency require raising turbine firing temperatures, which negatively impact component life. Under these conditions high strength materials are needed to provide long life at elevated temperatures.

The traditional method of determining the service life of gas turbine components employs practical experience, experimental data, and finite element analysis (FEA). The boundary conditions, temperature and stress gradients, environmental conditions, and time-dependent variations induce a number of damage mechanisms, which cause crack initiation and eventual rupture. Common practice in the gas turbine industry is to conduct high temperature mechanical testing on a sample-sized specimen over a period of time and use practical experience (from previous design) to extend empirical models to failure time. Creep rupture life is typically estimated using experience and secondary creep (i.e., steady state) modeling with temperature dependence. Unfortunately, this method neglects the effects of both strain-softening behavior (tertiary creep modeling) and tensile/compressive asymmetry. Not including these damage mechanisms creates both strain history and component life estimates of limited accuracy. Using a more advanced tertiary creep damage model with tensile/compressive asymmetry improves the ability to predict component life. Creep deformation and rupture experiments are used to determine the constants of the constitutive model. As a part of this investigation, the material model is evaluated and a numerical parametric study is carried out to determine the total strain experienced at various combinations of uni-axial stress, temperature, and time.

2 Tertiary Creep Damage Model

A method has been established for modeling tertiary creep at high temperatures. This method is based on the Norton power law for secondary creep, i.e.,

\[ \dot{\epsilon}_{cr} = A \bar{\sigma}^n \]

where \( A \) and \( n \) are the creep strain coefficient and exponent, respectively, and \( \bar{\sigma} \) is the von Mises effective stress. The coefficient \( A \) exhibits Arrhenian temperature dependence, e.g.,

\[ A = B \exp \left( -\frac{Q_{ct}}{RT} \right) \]

Here \( Q_{ct} \) is the activation energy for creep deformation, and \( T \) is the temperature measured in K. The term \( R \) is the universal gas constant. Both \( A \) and its temperature-independent counterpart, \( B \), have units MPa\(^{-n}\) h\(^{-1}\). The exponent \( n \) may also exhibit temperature dependence. As shown in Eq. (1), the creep strain rate is stress dependent due to \( \bar{\sigma} \). For most materials, traditional methods tend to find \( A \) and \( n \) to be stress independent but there are exceptions [1]. The secondary creep constants are determined by considering the minimum strain rate versus the applied stress, and using a regression analysis. Hyde et al. [2] showed for a Ni-based
alloy Waspaloy that once a critical value of applied stress is reached the relationship between minimum creep strain and stress evolves. This leads to a two-stage relationship and stress dependence of the secondary material constants. Table 1 shows that for the collected creep deformation data, only two stress levels per temperature were conducted. Any possible stress dependence of $A$ and $n$ is not visible in the current data. Therefore, stress dependence of the secondary material constants is neglected.

Jeong et al. [3] demonstrated that while under stress relaxation short term transient behavior occurs where $Q_m$ and $n$ are both stress dependent at relatively short times (e.g., less than 10 min). After this transient stage, the Norton power law becomes independent of initial stress and strain. This transient behavior has been neglected here since the current study focuses on longer periods (e.g., 1–100,000 h).

During the transition from secondary to tertiary creep, stresses in the vicinity of voids and microcracks along grain boundaries of polycrystalline materials are amplified. The stress concentration due to the local reduction in cross-sectional area is the phenomenological basis of the Kachanov–Rabotnov [4,5] damage variable, $\omega$, that is coupled with the creep strain rate and has a stress-dependent evolution, i.e.,

$$\dot{\epsilon}_{cr} = \dot{\epsilon}_{cr}(\bar{\sigma}, T, \omega, \ldots)$$

$$\dot{\omega} = \dot{\omega}(\bar{\sigma}, T, \omega, \ldots)$$

(3)

A straightforward formulation implies that the damage variable is the net reduction in cross-sectional area due to the presence of defects such as voids and/or cracks, e.g.,

$$\bar{\sigma}_{net} = \frac{\sigma - A_0}{A_{net}} = \frac{\bar{\sigma}}{1 - \omega}$$

(4)

where $\bar{\sigma}_{net}$ is called the net (area reduction) stress, $A_0$ is the undeformed area, $A_{net}$ is the deformed area due to deformation, and $\omega$ is the damage variable. Some models account for the variety of physically observed creep damage mechanisms with multiple damage variables [6]. An essential feature of damage required for application of continuum damage mechanics concepts is a continuous distribution of damage. Generally, microdefect interaction (in terms of stress fields, strain fields, and driving forces) is relatively weak until impingement or coalescence is imminent. By substituting Eq. (4) into Eq. (1), where $\bar{\sigma}$ is replaced by $\bar{\sigma}_{net}$, the steady-state creep strain is expressed as a modified secondary creep rate:

$$\dot{\epsilon}_{cr} = A\left(\frac{\bar{\sigma}}{1 - \omega}\right)^n$$

(5)

The isotropic damage variable is scalar valued from 0 for an initial state (e.g., undamaged) to 1 at final state (e.g., completely ruptured). Its evolution was given by Rabotnov [4] as

$$\dot{\omega} = \frac{M\bar{\sigma}^n}{(1 - \omega)^{m}}$$

(6)

where $M$ is the creep damage coefficient having units of MPa$^{-n}$. This damage evolution formulation allows the model to account for tertiary creep. In an earlier study, Kachanov [5] developed a similar formulation with the exception that the creep damage exponents, $\chi$ and $\phi$, were equivalent. Time hardening, tensile/compressive asymmetry, and multi-axial behavior can be accounted for in the model by replacing the effective stress with one that considers multi-axial stress, e.g.,

$$\dot{\omega} = \frac{M\sigma_r^n}{(1 - \omega)^{m}}$$

(7)

The exponent, $m$, accounts for time hardening and the units of $M$ are in MPa$^{-n} h^{-m}$; however, $m$ is defined to be 0.0 in most cases [7,8]. Here $\sigma_r$ describes the Hayhurst (tri-axial) stress and is related to the principal stress, $\sigma_1$, and hydrostatic (mean) stress, $\sigma_m$, and $\bar{\sigma}$. The Hayhurst stress [9] is given as

$$\sigma_r = \alpha\sigma_1 + 3\beta\sigma_m + (1 - \alpha - \beta)\bar{\sigma}$$

(8)

where $\alpha$ and $\beta$ are the weighing factors valued between 0 and 1 and are determined from multi-axial creep experiments. This formulation allows damage under uni-axial compression when $\alpha + 2\beta \approx 1$. More recent investigations have extended this damage evolution description to account for anisotropic damage resulting from multi-axial states of stress [10]. In these cases, damage is described by a second-rank tensor.

The coupled evolution (Eqs. (5) and (7)), which make up the tertiary creep damage model, can be reverted back to secondary creep when $M=0$. As a result, the damage evolution equation to model reverts back to the Norton power law for secondary creep.

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**Table 1 Creep Deformation data for IN617**

<table>
<thead>
<tr>
<th>Temperature, $T$</th>
<th>Stress, $\sigma$</th>
<th>Stress ratio, $\sigma/\sigma_{cr}$</th>
<th>Min strain rate, $\dot{\epsilon}$/h</th>
<th>Rupture strain, $\epsilon_c$ (%)</th>
<th>Time to strain, $t$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>°C / °F</td>
<td>MPa / ksi</td>
<td></td>
<td>0.25% strain</td>
<td>1% strain</td>
<td>5% strain</td>
</tr>
<tr>
<td>649 / 1200</td>
<td>310 / 45</td>
<td>1.22</td>
<td>0.0006</td>
<td>1.1</td>
<td>204</td>
</tr>
<tr>
<td>649 / 1200</td>
<td>310 / 45</td>
<td>1.22</td>
<td>0.0008</td>
<td>1.2</td>
<td>69</td>
</tr>
<tr>
<td>649 / 1200</td>
<td>310 / 45</td>
<td>1.36</td>
<td>0.0024</td>
<td>1.2</td>
<td>54</td>
</tr>
<tr>
<td>649 / 1200</td>
<td>310 / 45</td>
<td>1.36</td>
<td>0.0024</td>
<td>1.2</td>
<td>90</td>
</tr>
<tr>
<td>760$^o$</td>
<td>140 / 22</td>
<td>0.53</td>
<td>0.0098</td>
<td>16.0</td>
<td>7</td>
</tr>
<tr>
<td>760 / 1400</td>
<td>145 / 22</td>
<td>0.53</td>
<td>0.0051</td>
<td>19.8</td>
<td>14</td>
</tr>
<tr>
<td>760 / 1400</td>
<td>158 / 23</td>
<td>0.58</td>
<td>0.0031</td>
<td>13.3</td>
<td>19</td>
</tr>
<tr>
<td>760 / 1400</td>
<td>158 / 23</td>
<td>0.58</td>
<td>0.0035</td>
<td>14.0</td>
<td>16</td>
</tr>
<tr>
<td>871 / 1600</td>
<td>62 / 9</td>
<td>0.29</td>
<td>0.0022</td>
<td>22.0</td>
<td>54</td>
</tr>
<tr>
<td>871 / 1600</td>
<td>62 / 9</td>
<td>0.29</td>
<td>0.0022</td>
<td>22.0</td>
<td>54</td>
</tr>
<tr>
<td>871 / 1600</td>
<td>69 / 10</td>
<td>0.32</td>
<td>0.0082</td>
<td>19.3</td>
<td>17</td>
</tr>
<tr>
<td>871 / 1600</td>
<td>69 / 10</td>
<td>0.32</td>
<td>0.0088</td>
<td>28.5</td>
<td>14</td>
</tr>
<tr>
<td>982$^o$</td>
<td>1800 / 3.5</td>
<td>0.19</td>
<td>0.0002</td>
<td>19.0</td>
<td>170</td>
</tr>
<tr>
<td>982 / 1800</td>
<td>24 / 3.5</td>
<td>0.19</td>
<td>0.0002</td>
<td>14.5</td>
<td>228</td>
</tr>
</tbody>
</table>

$^a$Removed from consideration.
with the exception that $\bar{\sigma}$ is replaced by $\sigma_r$. This is a useful property that is exploited later to determine the transition from secondary to tertiary creep dominance over time.

The tertiary creep damage formulation can be applied to model the gradual reduction in load carrying capability (e.g., secondary through tertiary creep). Since $\omega$ characterizes the microstructural evolution of the material and is dependent on stress and temperature, the damage variable is an internal state variable (ISV). This formulation has been used in a variety of studies of turbine and rotor materials. The constants $A$, $n$, $M$, $\chi$, and $\phi$ are considered material properties. For example, in Waspaloy at 700°C, $A=9.23 \times 10^{-33}$ MPa$^{-n}$ h$^{-1}$, $n=10.65$, $M=1.18 \times 10^{-35}$ MPa$^{-3}$ h$^{-1}$, $\chi=8.13$, and $\phi=13.0$ [7]. For stainless steel at 650°C, $A=2.13 \times 10^{-13}$ MPa$^{-n}$ h$^{-1}$, $n=3.5$, $M=9.0 \times 10^{-10}$ MPa$^{-3}$ h$^{-1}$, and $\chi = \phi=2.8$ [8].

### Creep Experiments

Polycrystalline (PC) nickel-chromium-cobalt-molybdenum alloys are a modern class of materials commonly used in gas turbine components. One such material is wrought Ni-based alloy IN617. This nonproprietary sheet material has several equivalent monikers (e.g., Inconel-617, INCO-617, Nicrofer® 5520 Co-Alloy 617, and Haynes 617). The nominal chemical composition is given in Table 2. The alloying composition of the material allows it to withstand a range of conditions critical to gas turbine components. The combination of various elements produces a strong atomic structure that imparts strength at high temperatures. Although it is suitable below 1000°C, IN617 displays exceptionally high levels of creep rupture strength, even at temperatures of 980°C (1800°F). An inherent feature of the material is its high corrosion resistance compared with other standard Ni-based alloys such as alloy 556, alloy 230, and alloy Hastelloy-X [11]. It carries a good combination of resistance to oxidation and carburizing atmospheres that are important for components that undergo long term loading. Thermal shock resistance is another key feature that forward the uses of Ni-based materials [12]. All of these features result in a material that has good resistance to creep deformation. The material also has relatively good weldability allowing field repair; this is another important requirement of gas turbines. One major detractor for this material is that it is susceptible to intense nitridation at elevated temperatures while undergoing thermal cycling. This susceptibility is due to the development of crack defects in the oxide scale, which perpetuates the nitridation process and allows surface intrusion of oxides, thus leading to crack initiation and eventual failure [13]. Additionally, alloy 617 encounters deep internal defects through the development of aluminum nitrides, which severely hinder the load carrying capability of the material. Despite this condition, alloy 617 is still found to be the best combination of overall material properties. Proper turbine operation has been shown to produce gas turbine transition pieces of alloy 617 that last up to 15,000 total operating hours [14].

The tensile properties of the material are given in Table 3 [15]. An attractive property of alloy 617 is that it has a nonlinear yield strength, which is enhanced at temperatures between 700°C and 800°C [16]. This yield strength anomaly makes the material an excellent candidate for components that are heated within that temperature range (gas turbine transition pieces). This phenomenon has the added effect of making the creep deformation and rupture response at temperatures higher than 800°C more sensitive to temperature change (due a drop in yield strength).

Creep deformation and rupture experiments were conducted according to an ASTM standard E-139 [17] at 649°C, 760°C, 871°C, and 982°C on samples incised from wrought slabs of IN617. The combination of conditions is given in Table 1. From this variety of creep deformation and rupture experiments, the subject PC material exhibits primary, secondary, and tertiary creep strains depending on the combination of test temperature and stress, as shown in Fig. 1. Each test was conducted twice per stress level to account for variability in alloy samples due to slight differences in heat treatment. This method showed that inconsistencies are found at 760°C with 148 MPa and 982°C with 24 MPa, shown in Table 1. To account for these, these two creep curves were removed from the analysis.

### Based on these strain histories, the deformation mechanisms can be inferred from investigations that complemented mechanical experimentation with microscopic analysis [18,19]. Deformation at 649°C at high stress is mostly due to plastic strain, along with primary and secondary creeps. At stress levels of 345 and 310 MPa, the stress ratios, applied stress over yield strength, are found to be 1.35 and 1.22, respectively. Upon loading, a substantial time-independent plastic strain was produced before the creep test began. During primary creep, the strain rate, which is initially high, decreases significantly. This transient phenomenon is due to the ease at which dislocations can glide upon initial loading (due to initially present dislocations in untreated workpieces) and the saturation of the dislocation density, which inhibits continued creep deformation. Transient creep also drops to pinning of the dislocations when they encounter various obstacles after the easy glide period. Secondary creep arises due to the nucleation of grain boundary (GB) cavities and grain boundary sliding. The deformation mode of tertiary creep was not observed in these cases. At higher temperatures (760°C and above), deformation is dominated by elastic strain, and secondary and tertiary creeps are facilitated by the coalescence of grain boundary voids into microcracks. Sharma et al. [20] showed that at temperatures above 800°C, dynamic recrystallization is found to occur. This process is a part of the hot deformation process, which decreases average grain size as stress increases. This has the effect of rapidly reducing dislocation density in the alloy at higher temperatures and thus improving its deformation properties.

### Table 2: Nominal chemical composition of IN617

<table>
<thead>
<tr>
<th>Element</th>
<th>Cr</th>
<th>Co</th>
<th>Ti</th>
<th>Al</th>
<th>Mo</th>
<th>C</th>
<th>B</th>
<th>Fe</th>
<th>Ni</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>22.5</td>
<td>12.5</td>
<td>0.6</td>
<td>1.0</td>
<td>9.6</td>
<td>0.08</td>
<td>0.004</td>
<td>0.98</td>
<td>Bal.</td>
</tr>
</tbody>
</table>

### Table 3: IN617 material properties

<table>
<thead>
<tr>
<th>Temperature, $T$</th>
<th>0.2% tensile yield strength, $\sigma_y$</th>
<th>Tensile strength</th>
<th>Young’s modulus</th>
<th>Elongation</th>
</tr>
</thead>
<tbody>
<tr>
<td>°C</td>
<td>°F</td>
<td>MPa</td>
<td>ksi</td>
<td>MPa</td>
</tr>
<tr>
<td>649</td>
<td>1200</td>
<td>254</td>
<td>37</td>
<td>586.2</td>
</tr>
<tr>
<td>760</td>
<td>1400</td>
<td>272.7</td>
<td>40</td>
<td>525.5</td>
</tr>
<tr>
<td>871</td>
<td>1600</td>
<td>215.6</td>
<td>31</td>
<td>341</td>
</tr>
<tr>
<td>982</td>
<td>1800</td>
<td>128.4</td>
<td>19</td>
<td>181.6</td>
</tr>
</tbody>
</table>

*Yield strength assumed symmetric.*
reducing primary creep deformation. Notwithstanding, primary creep was still observed, but its contribution to the overall deformation was negligible compared with that of secondary and tertiary creeps; therefore for the case of this study primary creep was neglected. For all cases except those at 649°C, time-independent plasticity is ignored. Creep deformation in the current study is assumed to also encompass time-dependent viscoplasticity. Later studies will evaluate the response of a coupled plasticity and creep model.

The creep rupture data at 649°C, 760°C, 871°C, and 982°C compare well to an earlier comprehensive study [11] of creep rupture at temperature between 600°C and 1000°C, from previous studies [21–23]. Figure 2 shows stress versus rupture time of both the obtained and study creep rupture data. Generally, creep rupture time increases with either decreasing applied stress or decreasing temperature. The plot shows that the obtained creep rupture data are positioned within the bounds of creep rupture data at
adjacent temperature from the comprehensive study. It is important to note that in Fig. 2 the creep tests performed at 649°C or less experienced a time-independent plastic strain. In all the creep rupture data, scatter is present. This scatter is common in creep rupture charts. It is due to inconsistency in commercial material quality and inadequate control of the heat treatment process. It also is a result of differences in testing procedure and equipment. In the comprehensive study, multiple data sources were used where the testing procedure and equipment are not known. Manufacturers commonly use least-squares linear regression analysis to determine the minimum creep rupture life of an alloy while neglecting scatter. Advancements by Zuo et al. [24] show that the maximum likelihood method produces more accurate predictions of creep rupture life.

4 Numerical Methodology

The constitutive model described in Eqs. (5), (7), and (8) has been implemented in a general-purpose FEA software in order to determine the constants for the constitutive model used in the secondary-tertiary creep formulation. The formulation was implemented into a FORTRAN subroutine in the form of a user-programmable feature (UPF) in ANSYS [25]. The subroutine is incorporated with an implicit integration algorithm. This backward Euler integration algorithm is more accurate over long time periods than other practical numerical integration methods. This implementation allows for the update of the internal state variables. The subroutine is integrated with an implicit integration algorithm. This backward Euler integration algorithm is more accurate over long time periods than other practical numerical integration methods. This allows larger time steps that reduce the numerical solve time. Since the viscoplastic/creep behavior of materials is significant at extended histories, the backward Euler integration algorithm is the desired method for integration of creep constitutive models [26].

This implementation allows for the update of the internal state variable, \( \omega \). Initially, \( \omega = 0.0 \) and during loading, \( \omega \) increases. To prevent the singularity that is caused by rupture (e.g., \( \omega = 1.0 \)), the damage is restricted to a maximum of 0.90. Comparison of creep rupture data to numerical simulations shows that rupture occurs at \( \omega \) between 0.4 and 0.6; therefore, rupture can be achieved before damage reaches \( \omega = 1.0 \). To prevent an excessive model solve time and large deformation errors, a simulation was terminated once the total strain reached 100%. If needed, this model can be applied with time-independent and time-dependent plasticity models in a straightforward manner.

The UPF has been used to simulate the temperature and stress loading conditions of a series of uni-axial creep and rupture experiments. A single, solid, three-dimensional, eight-noded element was used, and the appropriate initial and boundary conditions were applied to numerically simulate the uni-axial creep deformation curves shown in Fig. 1. The loads and temperatures applied numerically simulated those of the obtained creep rupture experiments. Figure 3 demonstrates how these conditions are applied on the FEM geometry. The constant axial force load experience during tensile testing was applied as a force load on the top surface of the element. A uniform temperature was applied across the element. Displacement controls were set to match the constraints experienced during tensile testing. These conditions lead to an accurate assessment of the creep deformation within a single element model. Histories of creep deformation, \( \epsilon_{cr}(t) \), were recorded to a data file and subsequently compared with experiments. Using this method, expansion beyond a single element to multi-element simulations will produce an accurate measure of creep deformation.

A goal of the research was to provide a uni-axial creep deformation formulation that was capable of matching the material response up to 5% of the total strain determined from experiments. The time required for an experimentally tested specimen to deform to 5% is shown in Table 1.

The secondary creep constants (e.g., \( A \) and \( n \)) were evaluated directly from the available creep deformation and rupture data for IN617 at each temperature. As described earlier, \( A \) and \( n \) were found by plotting the minimum strain rate versus applied stress and using regression analysis in the form of Eq. (1) to determine the values of \( A \) and \( n \). Sharma et al. [20] found for IN617 an apparent activation energy, \( Q_{cr} \), equal to 271 kJ/mol similar to what has been observed in a bulk specimen tested in a He+O₂ environment [27]. Using this \( Q_{cr} \) value as an initial guess produced a \( B \) constant of 1.27 \( \times 10^{19} \) MPa⁻ⁿh⁻¹. A value of \( B \) that is less than 10⁻¹² is not suitable for application in FEM. Such numbers experience numerical cutoff where trailing significant figures are discarded resulting in limited accuracy. To overcome this problem, the guess value of \( Q_{cr} \) was doubled to 542 kJ/mol. Based on these constants and Eq. (2), \( B \) and \( Q_{cr} \) were approximated as 2.95 \( \times 10^{-3} \) MPa⁻ⁿh⁻¹ and 542 kJ/mol, respectively. The tertiary creep constants (e.g., \( M \), \( \chi \), and \( \phi \)) were determined for each temperature by an iterative trial and error method. This trial and error method consisted of parametrically changing each constant to determine its effect on the resulting creep deformation curve and manually refining the combined selection of constants until a best fit was determined. Further development using objective function based methods [28] can lead to better material constants, which match the creep deformation response up to creep rupture. For simplicity, both \( \chi \) and \( \phi \) were taken to be equivalent similar to what Altenbach et al. [8] considered for stainless steel. The constants \( \alpha \) and \( \beta \) were set to zero to produce a Hayhurst stress equivalent to uni-axially applied stress.

Coupling Norton’s law for secondary creep with the Kachanov–Rabotnov formulation for tertiary creep allows the finite element model to display a strain history that approximates the experimental results well. The strain histories simulated with the Norton–Rabotnov formulation have very good correlation with the analogous experimentally obtained \( \epsilon(t) \). Tertiary creep constants were optimized for every combination of temperature and stress. An example of the simulated creep deformation response is shown in Fig. 4. Temperature dependence is carried by the material constants while stress dependence is carried by the Hayhurst stress, \( \sigma_{cr} \). In each case, the trajectories of each of the correlated creep curves correspond to experimental data.

5 Temperature Dependence

All of the creep curves exhibiting creep induced strain softening can be generated by varying \( M \), \( \chi \), and \( \phi \). The creep damage rate coefficient, \( M \), displays temperature dependence; however, the damage rate exponents \( \chi \) and \( \phi \) were assumed as temperature independent. The constants at each temperature are plotted in Fig. 5. Based on the experiments, the temperature dependence for each of the constants is mathematically represented. The elastic modulus of IN617 was written as a third order polynomial, e.g.,
where $T$ are used in this case so that the minimum value of $n$ with $T$ is measured in K, and $M$ is measured in K. The McCauley brackets and $M_{\text{min}}$ term is compared with optimized constants for each temperature in Fig. 5. The square of the Pearson product-moment correlation coefficient, $r^2$, is valued at 5. This was implemented to ensure numerical convergence of the UPF at low temperatures. The creep damage coefficient is expressed as a two-parameter exponential power law function, e.g.,

$$M(T) = M_1 \exp(M_2T)$$

(11)

where $T$ is measured in K, and $M_1$ and $M_0$ are constants. The fit of Eqs. (9)–(11) is compared with optimized constants for each temperature in Fig. 5. The square of the Pearson product-moment correlation coefficient, $r^2$, indicates that each of the relations model the temperature dependence of the elastic and creep properties of IN617 with an accuracy greater than or equal to 0.90. The temperature dependence of the constants was coded into the FORTRAN subroutine so that predictions of creep deformation could be made through a range of temperatures.

### 6 Tensile/Compressive Asymmetry

It has been observed that for a number Ni-based alloys, there is a difference in the resulting mechanical response of a body undergoing compressive versus tensile loading [29–31]. This is due to the tendency for tensile loads to enhance the degradation of grain structure, which produces voids, crack propagation, and leads to rupture.Capturing the compressive behavior can be just as important as accurately modeling the tensile behavior, especially for materials used in gas turbine transition pieces. Transition pieces undergo multi-axial stress at high temperature, which can cause creep induced buckling [32]. An accurate estimate of service intervals could allow the preemptive patched repair maintenance of transition pieces, which has been found to extend the life of these components up to 8000 h [14]. To establish tensile/compressive asymmetric behavior in the constitutive model, a number of material constants were modified.

The weight constants $\alpha$ and $\beta$ in Eq. (8) modify Hayhurst stress and allow compression to occur when $\sigma+2\beta<1$. Hayhurst stress can be determined analytically for bi-axial stress states, as shown in Fig. 6. Hayhurst [33] showed that material constants of $\alpha=0$ and $\beta=0.25$ produce a comparable tensile/compressive asymmetry in Ni-based alloys. The state of stress with $\sigma_\alpha=\sigma$ and $\sigma_\beta=0$ yields a Hayhurst stress equal to $\sigma$. Applying the state of stress $\sigma_\alpha=-\sigma$ and $\sigma_\beta=0$ yields a Hayhurst stress that is $\sigma/2$. The constitutive model was modified to use this set of constants. It should be noted that the $\alpha$ and $\beta$ constants were not optimized for the IN617 alloy due to the lack of availability of compressive creep data.

Additional control of tensile/compressive asymmetry is provided in the damage rate equation (7) by the constants $\chi$ and $\phi$. Increasing the exponent $\chi$ increases the effect that the Hayhurst stress has on the damage rate. The constant $\phi$ modifies the effect that the existing damage has on the damage rate. The constant $\phi$ can have a profound effect particularly when the damage nears $\omega=1.0$. For this study $\chi=3.0$, based on guidance from other studies.

The tensile/compressive creep deformation response at 815.5°C and various stresses is shown in Fig. 7. This plot demonstrates that in terms of secondary creep, the material is modeled...
to exhibit symmetric tensile and compressive behaviors; however, as tertiary creep damage accumulates, the effects of void formation and coalescence are predicted to occur more rapidly under tensile conditions.

The tertiary creep model shows an overall reduction in the creep strain rate with respect to decreasing applied stress levels. This is attributed to the modified Hayhurst stress facilitating a smaller effective stress to the damage rate. Furthermore, due to the constants $\chi$ and $\phi$, the damage rate is modified causing a lower creep strain rate to be produced.

Using the tertiary creep model, Fig. 7 shows a factor of 4.7 in the time to 5% total strain between tension and compression at 100 MPa. At 10% total strain, this factor is increased to 5.5. The $\chi$ and $\phi$ constants have great significance due to the time dependence of damage and strain and the extensive amount of time required to reach rupture at lower stress levels. When applied stress is increased, the factors at 5% and 10% strain are reduced. This demonstrates that as applied stress is increased the tensile/compressive asymmetry found in the Ni-based alloy is reduced. Increased applied stress causes a reduction in rupture time. The secondary creep stage has less time to develop and in some cases is bypassed. This results in the tertiary creep stage becoming dominant earlier during creep deformation. The rapid deformation, which occurs during the tertiary creep stage, allows less time for tensile/compressive divergence. It should be emphasized that due to compressive creep data not being available for IN617, the compressive results could not be verified. The compressive results should, therefore, be used qualitatively.

7 Dominant Deformation Modes

One of the key goals of this research is to demonstrate the time dependence of the dominance of elasticity, secondary creep, and tertiary creep strain as stress and temperature change. The dominant deformation mode was determined by creating dominance contours. The ratio of partitioned strain over total strain for each stage was found. The partition with the largest ratio was considered dominant. The following equations were used to determine the curves of dominance transition:

$$ e_{\text{total}} = \frac{\sigma}{E} + e_{\text{cr}} = e_{\text{E}} + (e_{\text{SC}} + e_{\text{TR}}) $$

Fig. 7 Creep deformation predictions in tension and compression at 815.5°C for both secondary creep (dotted lines) and tertiary creep models (solid lines)
dominant mode = \max \left\{ \frac{e_E}{e_{total}}, \frac{e_{SC}}{e_{total}}, \frac{e_{TR}}{e_{total}} \right\} \quad (12)

where \( e_{total} \) is total strain, \( e_E \) is elastic strain, \( e_{SC} \) is secondary creep strain, and \( e_{TR} \) is tertiary creep strain. Based on this dominant deformation formulation, Fig. 8 shows the transition of the various deformation modes. Figure 8(a) depicts the time-dependent evolution of each form of strain at 815.5°C and 100 MPa. It clearly shows that the secondary creep model produces a linear strain-time function, which continues at the same rate until failure, while the tertiary creep model follows the expected exponential trend of an increase deformation rate, which leads to failure, while the tertiary creep model follows the expected exponential strain-time function, which continues at the same rate until failure.

Fig. 8 Comparison of ((a) and (c)) total strain and ((b) and (d)) dominant deformation mode for ((a) and (b)) 100 MPa and 815.5°C and ((c) and (d)) 250 MPa and 649°C.

…equivalent value of stress, a stress ratio, \( \sigma / \sigma_{YS} \), applied stress over yield strength was used. Figure 9 shows that at low stress the secondary and tertiary creep dominance lines converge. At some minimum stress, the difference between secondary and tertiary creeps becomes negligible. This minimum stress is temperature dependent. As applied stress approaches, the value of the yield strength deformation occurs more rapidly. When the stress ratio is very near unity, plastic deformation begins to occur. This demonstrates the need for a plasticity model to account for inelastic mechanical behavior.

Using temperature levels from 649°C to 982°C at 8.3°C intervals and stress levels from 500 MPa to 500 MPa at 25 MPa intervals, the simulations were carried out in an increasing magnitude of hours from 1 h to 100,000 h. The temperature range was restricted to 649–982°C to match the range used in experimental test. At lower temperatures, creep deformation disappears. The assumption is made that there is no stress dependence exhibited in the material constants such that two stress levels are sufficient to fit our material constants for the entire range of stresses. Stresses ranging from \( \pm 500 \) MPa are used to allow a clear picture of the relationship between stress and creep strain. It is particular helpful in making the large gradient of creep strain observed at low time periods (1 h, 10 h, and 100 h) visible. Simulations were conducted in elasticity only, secondary creep (e.g., \( M = 0 \)), and tertiary creep including tensile/compressive asymmetry conditions (20,172 simulations total).

With parametric boundary conditions established, a closer look at the response of the model can be achieved. Figure 10 shows the trends of creep stage dominance overlaid upon total strain for the complete set of simulations. It demonstrates the importance of implementing a tertiary creep model that includes effects neglected in a secondary creep model. In the figure, the region between the solid white lines signifies simulations where the elastic strain is dominant. The region between the solid white and black lines represents secondary creep strain dominance, and anything outside of the black lines produce tertiary creep strain dominance.
The dashed white and black line represents the 0.2% yield strength from Table 3. At stress levels above this line, a coupled plasticity and creep model is necessary to determine the total strain. Therefore, the simulated total strain found beyond this line should not be considered. The figure shows that at extended histories, creep strain becomes dominantly tertiary following expected experimentally reproducible trends. It should be noted at more than 1000 h under tensile loading, the secondary creep dominance begins to disappear. This is signified by the tertiary creep transition line passing the secondary dominance line in effect of a dominance shift. This is a result of mechanical load, time, and temperature producing a situation where the secondary creep stage is not strong enough to produce dominance. This results in tertiary creep becoming the primary source of creep deformation until failure. A similar trend is observed when stress is low and temperature is increasing; the secondary creep strain dominance transition line converges with the tertiary transition line.

The tensile/compressive relationship is demonstrated in Fig. 10 using the established $\alpha$, $\beta$, $\chi$, and $\phi$ constants. It shows that for compressive loading the tertiary creep transition line lies at higher stresses than tensile loading. This causes secondary creep to be dominant in a large range of stress and temperatures while under compression. This is a product of using the Hayhurst stress equation that produces a lower effective stress while under the same strain and the damage rate equation, which compounds these effects. Additionally the dominance shift noticed at 1000 h under tensile loading is delayed on the compressive side to 10,000 h. There is a significant difference in the creep dominance behavior of tensile/compressive asymmetric materials.

A major feature of the dominant deformation mode formulation is that it can be used as a design process tool for gas turbine components, as shown in Fig. 10. When considering new gas turbine component designs, it can be used as a method to eliminate boundary condition combinations leading to premature failure. The eliminated conditions could be determined based on yield strength, deformation mode, or allowable strain. Simple modifications could allow inclusion of additional criteria (e.g., tensile strength and compressive strength).

8 Conclusions

The materials used in gas turbine components deform by a number of modes. The focus of this investigation is modeling the...
Creep deformation of materials used to fabricate turbine parts that exhibit strain softening at high temperatures (i.e., above 649°C). Creep cavitation and void coalescence along and within the boundaries of grains lead to area-reduction transverse to the applied loading axis. This leads to a gradual loss of stress-carrying ability. The Kachanov–Rabotnov type creep damage formulation was used to account for this behavior found in creep deformation and rupture experiments carried out on the Ni-based alloy IN617.

To account for temperature dependence, phenomenological models were developed for several mechanical properties.

Fig. 10 Total strain deformation maps from creep simulations
damage formulation was modified and directly implemented into the constitutive model routine so that predictions of creep deformation could be made through a range of temperatures. The correlated curves show excellent agreement with trends from creep deformation experiments. Utilizing the Hayhurst (tri-axial) stress formulation and the damage rate equation, the model was shown to be able to produce tensile/compressive asymmetry. The deformation mode dominance over an extensive range of mechanical load, temperature, and time configurations was determined using a new dominant deformation formulation. The difference between tensile and compressive dominances was also determined.

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Nomenclature

A, B, n = secondary creep constants
M, X, φ, m = tertiary creep constants
α, β = weighing factors in the Hayhurst stress
Q_a = apparent activation energy
R = universal gas constant
e^c = creep strain
e^r = creep strain rate
σ = stress
σ_h = Hayhurst (tri-axial) stress
σ_f = first principal stress
σ_m = hydrostatic (mean) stress
σ_v = von Mises stress
ω = damage
ω^c = damage rate

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