Lecture 6
Static Strength
Stress Concentration Factors
Chapter 5 - Failures Resulting from Static Loading

Strength is a property or characteristic of a mechanical element.

In mass-produced parts, Strength will vary due to variations in dimensions, machining, forming, and material manufacturing and composition.

Strength is statistical in nature, involving parameters such as the mean, standard deviation, and distribution identifiers.

\[
S_i, \quad i = \begin{cases} 
\text{allowable stress (i.e. design requirements)} \\
\text{yield strength} \\
\text{elastic - plastic boundary} \\
\text{ultimate tensile strength} \\
S_{u.t.s.} \quad S_{t.s.} / \text{rupture}
\end{cases}
\]

![Graphs showing tensile properties of P91 steel](image)

Tensile properties of P91 steel

- Mean: \( \bar{x} = \frac{1}{n} \sum x_i \)
- Standard Deviation: \( s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} \)
- Coefficient of Variation: \( \text{CoV} = \frac{s_x}{\bar{x}} \times 100 \)

![Probability Distribution Function](image)
Static Load is a stationary force or couple moment applied to a member. It is unchanged in magnitude and direction. It can exists as tensile, compressive, shear, bending, and torison loads or any combination of these.

![Diagram showing stress analysis](image)

\[
\sigma(x, y, z) = \begin{bmatrix}
\sigma_{x} \\
\sigma_{y} \\
\sigma_{z} \\
\tau_{xy} \\
\tau_{yz} \\
\tau_{zx}
\end{bmatrix} \rightarrow \begin{bmatrix}
\sigma_{1} \\
0 \\
0 \\
0 \\
0 \\
\sigma_{3}
\end{bmatrix}
\]

Stress can be unique at every point in a body!!!

Failure.

Failure can mean a part has seperated completely, has become permanently distorted, its reliability downgraded, had its function compromised, exceeded its safety factor, whatever the reason.

We as designers must decide what metric we will use for failure. It can mean any or all of those above.
Leaving the combustor, the hot exhaust is passed through the turbine (c in Fig. 1), in which the gases are partially expanded through alternate stator and rotor rows. Depending on the engine type, the turbine may consist of one or several stages. Like the compressor, the turbine is divided into low-pressure and high-pressure sections, the latter being closer to the combustor.

Based on the aforementioned observations, it was concluded that the blade failed as a result of crack propagation under low stresses of vibrations. The origin of the fatigue crack was a preliminary crack (or flaw), formed as a result of advanced hot corrosion attack.
Analysis regions on the blade A–A transversal section

Analysis regions on the blade convex side.

Blade airfoil stress distribution in the cutting plane at 60% vane height.

Termomechanical fatigue cracks in the cooling holes.

Figure shows coating deterioration in internal cooling holes of the airfoil. Loss of the coating constituents due to oxidation can be seen.

The cracks in the internal cooling holes in the hottest sections of the blade (airfoil central sections) were detected. Cracks initiate in the cooling hole coating and propagate into the substrate following grain boundary trajectories. The crack size reaches 0.4 mm. At the crack tip, small creep voids were identified on the intergranular facets at the extreme tip of the crack.

On the basis of these evidences it was evaluated that the crack initiation/propagation was derived by a mixed fatigue/creep mechanism. The coating crack initiation was probably due to a thermal fatigue mechanism as a result of high thermal transient loads (trips, start-ups and slow-downs), and crack grain boundary initiation and propagation in the substrate by a creep mechanism (high steady state load).

5-1 Static Strength

Ideally, in designing any machine element, the engineer should have available the results of a great many Strength Tests of the particular material chosen.

The tests should be of the material under the same boundary conditions (load, temperature, environment) it would experience in service.

The cost of conducting these tests is justified if failure of the part may endanger human life, lead to catastrophic failure of a mechanical system, or cause costly massive recalls.

When the risk of failure is low and/or not costly, the use of previously published data is allowed. (See Appendix A).

Many Material Property Handbooks exist for this purpose.

Examples of organizations with handbooks include:
ASM International
Society of Automotive Engineering
Air Force Office of Scientific Research
America Society of Mechanical Engineers
etc.
5-2 Stress Concentration

Stress can become high concentrated at localization within a machine part.

A discontinuity in a machine part alters the stress distribution in its vicinity.

Such discontinuities are called stress raisers, and the regions in which they occur are called stress concentrations.

- Shafts with shoulders and key slots
- Bolts heads and threads
- Holes, oil grooves, notched, and various other geometry
- Surface defects due to machine or mishandling

A theoretical, or geometric, stress concentration factor $K_{+3}K_{s}$ is used to relate the actual maximum stress at the discontinuity to the nominal stress.

Normal SCF

$$K_{f} = \frac{\sigma_{\text{max}}}{\sigma_{0}}$$

$\sigma_{0}$ - nominal normal stress

Shear SCF

$$K_{s} = \frac{\tau_{\text{max}}}{\tau_{0}}$$

$\tau_{0}$ - nominal shear stress

$\sigma_{\text{max}}$ or $\tau_{\text{max}}$ is the maximum stress that develops at the notch.

$\sigma_{0}$ or $\tau_{0}$ is the nominal stress calculated using the net area near the notch.

$K_{f}$ is the ratio of the two stresses.

The stress concentration factor depends only on the geometry and loading condition of the part. That is, the particular material used has no effect on the value.
Stress Concentration Factor Charts

**Example A**
Plate, tension, fillet at d by center hole

\[ k_1 = 2.5 \]

\[ F \]

\[ k_1 = 2.5 \]

**Example B**
Cylinder, shoulder, fillet, radius, bending

\[ k_4 = 1.8 \]

\[ M \]

\[ D \]

Appendix A-15 - Stress Concentration Factor Charts

Calculating Nominal Stress

Nominal Stress is calculated using the net area near the notch. In geometry where the area changes from one side of the notch to another, choose the small area.

**Example A**

\[ \sigma_0 = \frac{F}{(w-d)l} \]

**Example B**

\[ \sigma_0 = -\frac{M(d)}{I} \]
Stress concentration factor charts for many geometry and loading configurations are available in Handbooks.

The most important source of stress concentration factors is R.E. Peterson "Design Factors for Stress Concentration".

Techniques to reduce stress concentrations in a shoulder supported bearing with a sharp radius

(a) large radius undercut into the shoulder 
(b) large radius relief groove into the back of the shoulder 
(c) large radius relief groove into the small diameter.

Ductile Material Exception!

If a material is ductile and the load static, the design load may cause yielding in the critical location in the notch.

The material will strain-harden and the yield strength increase at the stress concentration.

The part can carry the load satisfactorily and failure will not occur.

In this case we set

$$K_+ = K_{\text{yield}} = 1$$

When using this rule for ductile materials, we must be careful to make sure the material is not susceptible to brittle failure in the environment of use.

Cast Iron Exception!

Cast Iron will brittle does not exhibit a stress concentration

$$K_+ = K_{\text{yield}} = 1$$
Examples
Example 3-13

The 2mm-thick bar is loaded axially to 10 kN. The bar material has been heat treated to raise its strength, but as a consequence it has lost most of its ductility. It is desired to drill a hole through the center of the 40-mm face to allow a cable to pass through. A 4-mm hole is sufficient for the cable to fit, but an 8-mm drill is readily available. Will a crack be more likely to initiate at the larger hole, the smaller hole, or at the fillet?

Knowns and Unknowns

Load
Dimensions
Constraints

\[ \sigma_{\text{max}} = \begin{cases} 4\text{-mm hole} \\ 8\text{-mm hole} \\ \text{fillet} \end{cases} \]

SCF Charts

(Appendix A-15)

Tensile finite width contoured circular hole

\[ d = \frac{4\text{mm}}{40\text{mm}} = 0.1 \]

\[ K_t \approx 2.7 \]

\[ d = \frac{8\text{mm}}{40\text{mm}} = 0.2 \]

\[ K_t \approx 2.5 \]

Tensile shelled bar

\[ D = \frac{40\text{mm}}{34\text{mm}} = 1.18 \]

\[ r = \frac{10\text{mm}}{34\text{mm}} = 0.029 \]

\[ K_t \approx 2.5 \]

Note: Caution!
Evaluate 4 mm hole

\[ k_+ = \frac{\sigma_{\text{max}}}{\sigma_0} \quad k_+ = \, , \sigma_0 = \, ? \]

**Nominal Stress** \( \sigma_0 \)

\[ \sigma_0 = \frac{F}{A_0} = \frac{F}{(w-d) \cdot t} = \frac{10,000 \text{N}}{(40-8) \cdot 2 \text{ mm}} = 139 \text{ MPa} \]

**SCF, \( k_+ \)**

Use the chart! \( \Rightarrow \) \( k_+ = 2.7 \)

\[ \sigma_{\text{max}} = k_+ \cdot \sigma_0 = 380 \text{ MPa} \]

Evaluate 8 mm hole

**Nominal Stress** \( \sigma_0 \)

\[ \sigma_0 = \frac{F}{A_0} = \frac{F}{(w-d) \cdot t} = \frac{10,000 \text{N}}{(40-8) \cdot 2 \text{ mm}} = 156 \text{ MPa} \]

**SCF, \( k_+ \)**

Use the chart! \( \Rightarrow \) \( k_+ = 2.5 \)

\[ \sigma_{\text{max}} = k_+ \cdot \sigma_0 = 390 \text{ MPa} \]

*The 8 mm hole leads to a higher stress concentration. Avoid using the 8 mm drill.*
Evaluate 1mm filet

\[
k_+ = \frac{\sigma_{\text{max}}}{\sigma_0}, \quad k_+ = \text{?} \quad \sigma_0 = \text{?}
\]

Normal stress \(\sigma_0\)

\[
\sigma_0 = \frac{F}{A_0} = \frac{F}{d \cdot t} = \frac{10,000 \text{N}}{34.2 \text{ mm}} = 147 \text{ MPa}
\]

SCF, \(k_+\)

Use the chart! \(\rightarrow k_+ \approx 2.5\) Comment!

\[
\sigma_{\text{max}} = k_+ \cdot \sigma_0 = 368 \text{ MPa}
\]

4 mm hole \(\sigma_{\text{max}} = 380 \text{ MPa}\)

8 mm hole \(\sigma_{\text{max}} = 320 \text{ MPa}\)

1 mm fillet \(\sigma_{\text{max}} = 368 \text{ MPa}\)

Most likely to crack in the following order

1. 8 mm hole \(\rightarrow\) Do not use!
2. 4 mm hole
3. 1 mm fillet (comment)