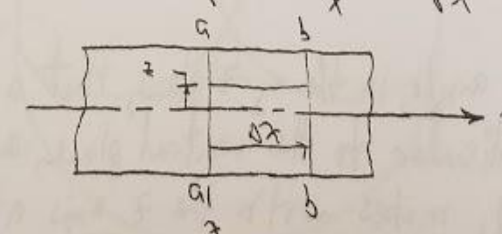
3.2).

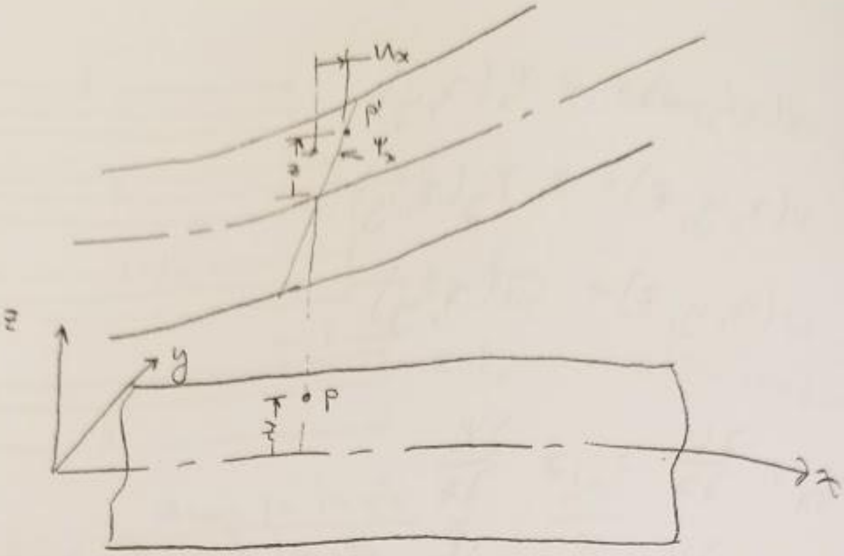
$$u(x,y,z) = z\,\psi_x(x,y)$$
$$v(x,y,z) = z\,\psi_y(x,y)$$
$$w(x,y,z) = \bar{w}(x,y)$$

$$\epsilon_x = \frac{\partial u}{\partial x} = z\frac{\partial \psi_x}{\partial x}$$
$$\epsilon_y = \frac{\partial v}{\partial y} = z\frac{\partial \psi_y}{\partial y}$$
$$\epsilon_z = 0$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = z\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right)$$
$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \psi_y + \frac{\partial \bar{w}}{\partial y}$$
$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \psi_x + \frac{\partial \bar{w}}{\partial x}$$




Assume $\bar{w}(x,y) = 0$ (i.e. shear applied oppositely on upper and lower faces):




From the geometry, assuming small displacements,

$$u_x = z\,\Psi_x$$

$\Psi_x$ is the angle, in the x-z plane, that a straight line perpendicular to the neutral plane, and passing through point P, makes with the z axis after deformation.

$\Psi_y$ is the angle, in the y-z plane, that a straight line perpendicular to the neutral plane, and passing through point P, makes with the z axis after deformation.

Problem 3-3

The problem states that:

$$\varepsilon_{yy} = -\nu\cdot\varepsilon_{xx} \qquad \gamma_{xy} = 0$$

The normal strain in the x' direction is, from the first of Eqs. (3-30),

$$\varepsilon_{x'x'} = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} + \frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\cdot\cos(2\cdot\theta) + \frac{1}{2}\cdot\gamma_{xy}\cdot\sin(2\cdot\theta)$$

Therefore, on substituting and simplifying,

$$\varepsilon_{x'x'}(\theta) = \varepsilon_{xx}\cdot\left[\frac{1}{2}\cdot(1-\nu) + \frac{1}{2}\cdot(1+\nu)\cdot\cos(2\cdot\theta)\right]$$

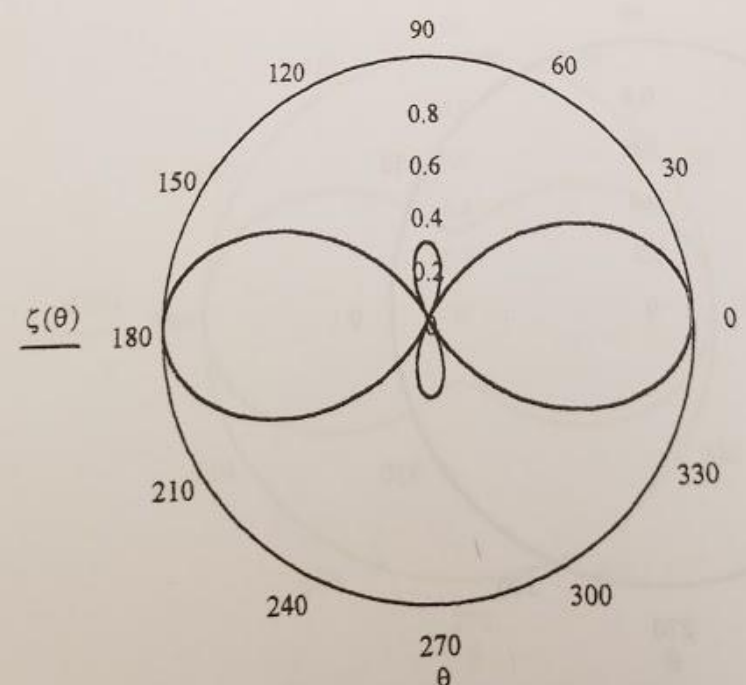
or,

$$\zeta = \frac{\varepsilon_{x'x'}}{\varepsilon_{xx}} = \frac{1}{2}\cdot(1-\nu) + \frac{1}{2}\cdot(1+\nu)\cdot\cos(2\cdot\theta)$$

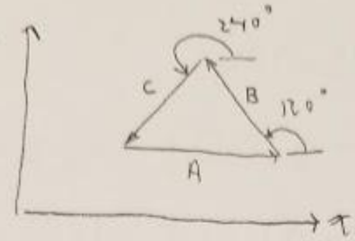
This can be plotted for the value of Poisson's ratio given:

$$\nu := .283$$

$$\zeta(\theta) := \frac{1}{2}\cdot(1-\nu) + \frac{1}{2}\cdot(1+\nu)\cdot\cos(2\cdot\theta)$$



#3-6




in the first of Eqs (3-30), with $\theta = 0$ and $\epsilon_{x'x'} = \epsilon_A$,

(1) $$\epsilon_A = \epsilon_{xx}$$

This is now repeated with $\theta = 120°$ and $\epsilon_{x'x'} = \epsilon_B$:

$$\epsilon_B = \frac{\epsilon_{xx}+\epsilon_{yy}}{2} + \frac{\epsilon_{xx}-\epsilon_{yy}}{2}\cos(240°) + \frac{1}{2}\gamma_{xy}\sin(240°)$$

$$\epsilon_B = \frac{\epsilon_{xx}+\epsilon_{yy}}{2} - \frac{\epsilon_{xx}-\epsilon_{yy}}{2}\cos(60°) - \frac{1}{2}\gamma_{xy}\sin(60°)$$

(2) $$\epsilon_B = \frac{1}{4}\epsilon_{xx} + \frac{3}{4}\epsilon_{yy} - \frac{\sqrt{3}}{4}\gamma_{xy}$$

Finally, with $\theta = 240°$, $\epsilon_{x'x'} = \epsilon_C$,

$$\epsilon_C = \frac{\epsilon_{xx}+\epsilon_{yy}}{2} + \frac{\epsilon_{xx}-\epsilon_{yy}}{2}\cos(480°) + \frac{1}{2}\gamma_{xy}\sin(48$$

$$\epsilon_C = \frac{\epsilon_{xx}+\epsilon_{yy}}{2} - \frac{\epsilon_{xx}-\epsilon_{yy}}{2}\sin(30°) + \frac{1}{2}\gamma_{xy}\cos(3$$

(3) $$\epsilon_C = \frac{1}{4}\epsilon_{xx} + \frac{3}{4}\epsilon_{yy} + \frac{\sqrt{3}}{4}\gamma_{xy}$$

Subtract Eq(2) from Eq(3):

$$\epsilon_C - \epsilon_B = \frac{\sqrt{3}}{2}\gamma_{xy}$$

(4) $\therefore \gamma_{xy} = \frac{2}{\sqrt{3}}(\epsilon_C - \epsilon_B)$

Add Eqs(2) and (3), and use Eq(1) for $\epsilon_{xx}$:

$$\epsilon_B + \epsilon_C = \frac{1}{2}\epsilon_A + \frac{3}{2}\epsilon_{yy}$$

(5) $\therefore \epsilon_{yy} = \frac{2}{3}\left(-\frac{1}{2}\epsilon_A + \epsilon_B + \epsilon_C\right) = \frac{1}{3}\left(-\epsilon_A + 2\epsilon_B + 2\epsilon_C\right)$

Thus, $\epsilon_{xx} = \epsilon_A$

$$\epsilon_{yy} = \frac{2}{3}\left(-\frac{1}{2}\epsilon_A + \epsilon_B + \epsilon_C\right)$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\epsilon_C - \epsilon_B)$$

\# 3-7 From Eqs (3-30),

$$\epsilon_{x'x'} = \frac{\epsilon_{xx}+\epsilon_{yy}}{2} + \frac{\epsilon_{xx}-\epsilon_{yy}}{2}\cos(2\beta) + \frac{1}{2}\gamma_{xy}\sin(2\beta)$$

$$\epsilon_{y'y'} = \frac{\epsilon_{xx}+\epsilon_{yy}}{2} - \frac{\epsilon_{xx}-\epsilon_{yy}}{2}\cos(2\beta) - \frac{1}{2}\gamma_{xy}\sin(2\beta)$$

$$\gamma_{x'y'} = -(\epsilon_{xx}-\epsilon_{yy})\sin(2\beta) + \gamma_{xy}\cos(2\beta)$$

From problem 3-6,

$$\epsilon_{xx} = \epsilon_A$$

$$\epsilon_{yy} = \frac{2}{3}\left(-\frac{1}{2}\epsilon_A + \epsilon_B + \epsilon_C\right)$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\epsilon_C - \epsilon_B)$$

$$\therefore \epsilon_{xx} + \epsilon_{yy} = \frac{2}{3}(\epsilon_A + \epsilon_B + \epsilon_C)$$

$$\epsilon_{xx} - \epsilon_{yy} = \frac{2}{3}(2\epsilon_A - \epsilon_B - \epsilon_C)$$

Thus,

$$\epsilon_{x'x'} = \frac{\epsilon_A+\epsilon_B+\epsilon_C}{3} + \frac{2\epsilon_A-\epsilon_B-\epsilon_C}{3}\cos(2\beta) + \frac{\epsilon_C-\epsilon_B}{\sqrt{3}}\sin(2\beta)$$

$$\epsilon_{y'y'} = \frac{\epsilon_A+\epsilon_B+\epsilon_C}{3} - \frac{2\epsilon_A-\epsilon_B-\epsilon_C}{3}\cos(2\theta) - \frac{\epsilon_C-\epsilon_B}{\sqrt{3}}\sin(2\beta)$$

$$\gamma_{x'y'} = -\frac{2}{\sqrt{3}}(2\epsilon_A - \epsilon_B - \epsilon_C)\sin(2\beta) + \frac{2}{\sqrt{3}}(\epsilon_C - \epsilon_B)\cos(2\beta)$$