3.2 By Table A.1, for aluminum alloy 7075 T6, $E = 72\,GPa$, $\nu = 0.33$. Also, by Eq. (3.32), the principal stress–principal strain relations are

$$\sigma_1 = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_1 + \nu\epsilon_2 + \nu\epsilon_3\right] \qquad (a)$$

$$\sigma_2 = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\epsilon_1 + (1-\nu)\epsilon_2 + \nu\epsilon_3\right] \qquad (b)$$

$$\sigma_3 = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\epsilon_1 + \nu\epsilon_2 + (1-\nu)\epsilon_3\right] \qquad (c)$$

Since there is no normal pressure on the surface of the wing (the wing is being bent), $\sigma_3 = 0$. Hence, Eq. (c) yields the principal strain ϵ_3 as

$$\epsilon_3 = -\frac{\nu}{(1-\nu)}(\epsilon_1 + \epsilon_2) \qquad (d)$$

(cont.)

3.2 cont. Substituting Eq. (d) into Eqs. (a) and (b), we find

$$\sigma_1 = \frac{E}{1-\nu^2}(\epsilon_1 + \nu\epsilon_2), \quad \sigma_2 = \frac{E}{1-\nu^2}(\nu\epsilon_1 + \epsilon_2) \quad (e)$$

For $E = 72$ GPa and $\nu = 0.33$, Eqs (e) and (d) become

$$\sigma_1 = (80.80\epsilon_1 + 26.66\epsilon_2)\times 10^9 \ [\text{Pa}]$$
$$\sigma_2 = (26.66\epsilon_1 + 80.80\epsilon_2)\times 10^9 \ [\text{Pa}] \quad (f)$$

and

$$\epsilon_3 = -0.4925(\epsilon_1 + \epsilon_2) \quad (g)$$

For point 1, Table P3.2, $\epsilon_1 = -0.004$ and $\epsilon_2 = -0.006$. Then, Eqs. (f) and (g) yield

$$\sigma_1 = -483.2 \text{ MPa}, \ \sigma_2 = -591.4 \text{ MPa}, \ \epsilon_3 = 0.0049.$$

Similarly for points 2, 3, 4, and 5, we have

Point 2: $\epsilon_1 = 0.008$, $\epsilon_2 = 0.002$

$$\sigma_1 = 699.7 \text{ MPa}, \ \sigma_2 = 374.9 \text{ MPa}, \ \epsilon_3 = -0.0049.$$

Point 3: $\epsilon_1 = 0.006$, $\epsilon_2 = 0.002$

$$\sigma_1 = 538.1 \text{ MPa}, \ \sigma_2 = 321.6 \text{ MPa}, \ \epsilon_3 = -0.0039$$

Point 4: $\epsilon_1 = -0.005$, $\epsilon_2 = -0.008$

$$\sigma_1 = -617.3 \text{ MPa}, \ \sigma_2 = -779.7 \text{ MPa}, \ \epsilon_3 = 0.0064$$

Point 5: $\epsilon_1 = 0.002$, $\epsilon_2 = -0.002$

$$\sigma_1 = 108.3 \text{ MPa}, \ \sigma_2 = -108.3 \text{ MPa}, \ \epsilon_3 = 0$$

3.3

Since $\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = \sigma_{xy} = \epsilon_{yy} = 0$,

$$\epsilon_{yy} = 0 = \frac{1}{E}\sigma_{yy} - \frac{\nu}{E}\sigma_{xx}$$

$$\sigma_{yy} = \nu\sigma_{xx} = 0.29(500) = \underline{145.0 \text{ MPa}}$$

$$\epsilon_{xx} = \frac{1}{E}\sigma_{xx} - \frac{\nu}{E}\sigma_{yy} = \frac{500}{200{,}000} - \frac{0.29(145)}{200{,}000} = \underline{0.002290}$$

$$\epsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy}) = -\frac{0.29(500+145)}{200{,}000} = \underline{-0.000935}$$

$$L_x = 800(1.002290) = \underline{801.83 \text{ mm}}$$

$$L_y = \underline{800 \text{ mm}}$$

$$L_z = 10(1 - 0.000935) = \underline{9.99 \text{ mm}}$$

3.6 Initially the undeformed unit cube has volume $V_0 = 1$

The volume of the deformed cube is

$$V_1 = (1+\epsilon_{xx})(1+\epsilon_{yy})(1+\epsilon_{zz})$$

$$= 1 + \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} + \text{higher order terms}$$

The change in volume is, with Eq. (3.30)

$$\Delta V = V_1 - V_0 = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = \frac{1-2\nu}{E}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

Therefore, $\Delta V = 0$ for $\sigma_{xx} + \sigma_{yy} + \sigma_{zz} = 0$; $\nu \neq \frac{1}{2}$.

Note: If $\nu = \frac{1}{2}$, $\Delta V = 0$ for all values of $\sigma_{xx} + \sigma_{yy} + \sigma_{zz}$.

3.10

From Prob. (2.78), $\gamma_{xy} = 2\epsilon_b - \epsilon_a - \epsilon_c$. With $\epsilon_a = 0.00250\ \epsilon_{xx}$ $\epsilon_b = 0.00140$, $\epsilon_c = -0.00125 = \epsilon_{yy}$, $\gamma_{xy} = 0.00155$. By Table 1, Appendix A, $E = 72.0$ GPa, $\nu = 0.33$.

(a) Since $\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$, Eqs. (3.32) reduce to

$$\sigma_{xx} = \frac{E}{1-\nu^2}(\epsilon_{xx} + \nu\epsilon_{yy}),\ \sigma_{yy} = \frac{E}{1-\nu^2}(\epsilon_{yy} + \nu\epsilon_{xx}),\ \sigma_{xy} = G\gamma_{xy}.$$

Hence,
$$\sigma_{xx} = \frac{72{,}000}{1-0.33^2}\left[0.0025 + 0.33(-0.00125)\right] = 168.67 \text{ MPa}$$

$$\sigma_{yy} = \frac{72{,}000}{1-0.33^2}\left[-0.00125 + 0.33(.00250)\right] = -34.34 \text{ MPa}$$

$$\sigma_{xy} = \frac{72{,}000}{2(1+0.33)}(0.00155) = 41.95 \text{ MPa}$$

By Eqs. (2.37),

$$\sigma_1 = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) + \sqrt{\frac{1}{4}(\sigma_{xx} - \sigma_{yy})^2 + \sigma_{xy}^2}$$

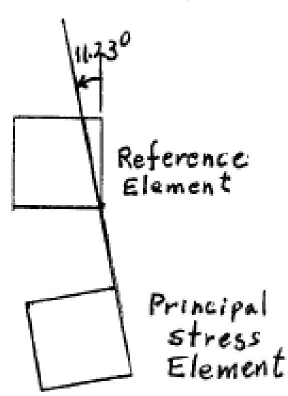
$$= \frac{1}{2}(168.67 - 34.34) + \sqrt{\left(\frac{168.67 + 34.34}{2}\right)^2 + 41.95^2}$$

$$= 67.16 + 109.83 = 176.99 \text{ MPa}$$

$$\sigma_2 = 67.16 - 109.83 = -42.67 \text{ MPa}$$

(b) $$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} = 0.4133;\ 2\theta = 0.3919 \text{ rad} = 22.45^\circ$$

$\therefore\ \theta = 11.23^\circ$



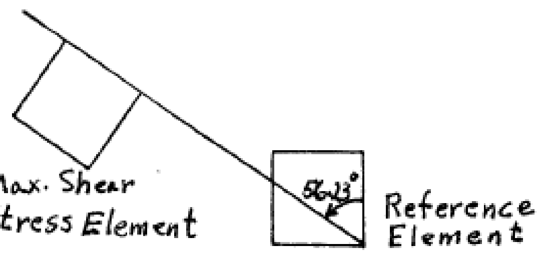
(Cont.)

3.10 continued

(c) $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{2}(176.99 + 42.67)$

or $\tau_{max} = 109.83$ MPa.

(d) θ for $\tau_{max} = 11.23° + 45° = 56.23°$



3.22 As in Problem 3.21, the z axis is a principal axis for both stress and strain.

(a) In the (x, y) plane, for stress, the angle of rotation from (x, y) axes to principal axes is, with $\sigma_{xx} = 7$ MPa, $\sigma_{yy} = 2.1$ MPa, $\sigma_{xy} = 1.4$ MPa

$$\theta_{stress} = \frac{1}{2}\arctan\frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} = \frac{1}{2}\arctan\frac{2(1.4)}{7 - 2.1} = 14.87° \quad \text{(a)}$$

Counterclockwise

(Cont.)

3.22 cont. (b). With $E_x = 15{,}920$ MPa, $E_y = 1195$ MPa, $E_z = 765$ MPa,

$\nu_{xy} = 0.426$, $\nu_{xz} = 0.451$, $\nu_{yz} = 0.697$, (b)

Eqs. (3.52) yield

$$\frac{\nu_{yx}}{E_y} = \frac{\nu_{xy}}{E_x} = \frac{0.426}{15{,}290} = 2.786 \times 10^{-5}, \quad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} = \frac{0.451}{15{,}290} = 2.950 \times 10^{-5}$$

$$\frac{\nu_{zy}}{E_z} = \frac{\nu_{yz}}{E_y} = \frac{0.697}{1195} = 5.833 \times 10^{-4} \qquad (c)$$

With Eqs(b) and (c) and $\sigma_{xx} = 7$ MPa, $\sigma_{yy} = 2.1$ MPa, $\sigma_{zz} = 2.8$ MPa, $\sigma_{xy} = 1.4$ MPa, $G_{xy} = 1130$ MPa, $G_{xz} = 1040$ MPa, $G_{yz} = 260$ MPa and $\sigma_{xz} = \sigma_{yz} = 0$, Eqs. (3.51) yield

$$\epsilon_{xx} = \frac{1}{E_x}\sigma_{xx} - \frac{\nu_{yx}}{E_y}\sigma_{yy} - \frac{\nu_{zx}}{E_z}\sigma_{zz} = \frac{7}{15290} - (2.786\times10^{-5})(2.1) - (2.95\times10^{-5})(-2.8) = 0.0004819$$

$$\epsilon_{yy} = -\frac{\nu_{xy}}{E_x}\sigma_x + \frac{1}{E_y}\sigma_{yy} - \frac{\nu_{zy}}{E_z}\sigma_{zz} = -(2.786\times10^{-5})(7) + \frac{2.1}{1195} - (5.833\times10^{-4})(-2.8) = 0.003195$$

$$\epsilon_{zz} = -\frac{\nu_{xz}}{E_x}\sigma_x - \frac{\nu_{yz}}{E_y}\sigma_{yy} + \frac{1}{E_z}\sigma_{zz} = -(2.95\times10^{-5})(7) - (5.833\times10^{-4})(2.1) + \frac{(-2.1)}{765} = -0.005092 \qquad (d)$$

$$\gamma_{xy} = \frac{1}{G_{xy}}\sigma_{xy} = \frac{1.4}{1130} = 0.001239$$

$$\gamma_{xz} = \gamma_{yz} = 0$$

(c) With Eqs (d), the angle of rotation in the (x,y) plane from the (x,y) axes to the principal axes is

$$\theta_{strain} = \frac{1}{2}\arctan\frac{\gamma_{xy}}{\epsilon_{xx} - \epsilon_{yy}} = \frac{1}{2}\arctan\frac{0.001239}{0.0004819 - 0.003195}$$

or

$$\theta_{strain} = -12.27^\circ \ (12.27^\circ \text{ clockwise}) \qquad (e)$$

Comparison of Eqs. (a) and (e) shows that

$$\theta_{stress} \neq \theta_{strain}$$