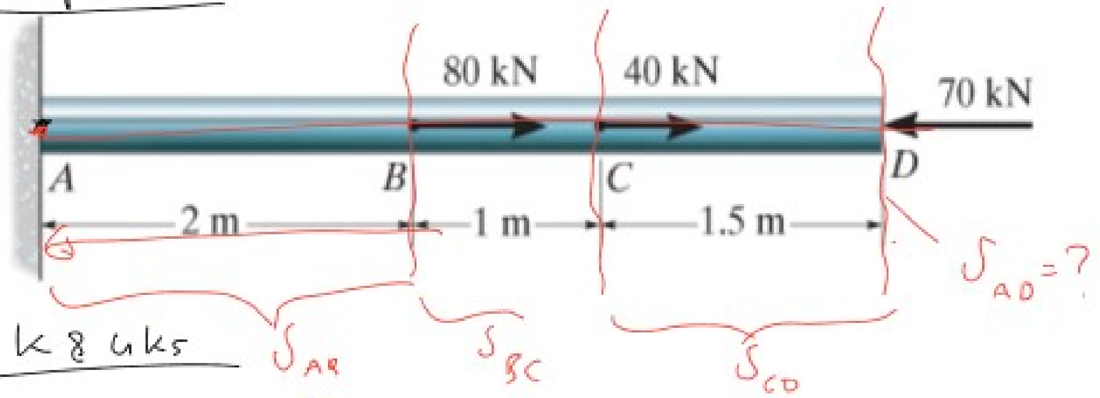
Lecture 1 Examples

Example 1



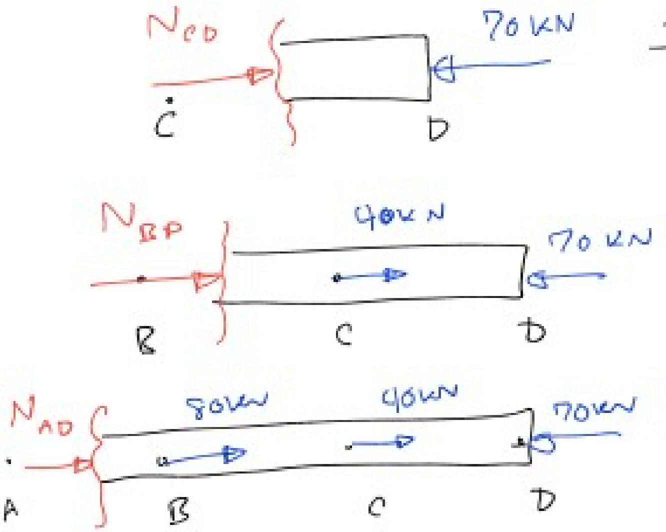
K & Uks

K : 70 kN, 40 kN, 80, d = 50 mm
Dimensions
A-36 Steel

Uks: $\delta_D = ?$, $\delta_{BC} = ?$, $N_{AB} = ?$

$$\delta = \sum \frac{P \cdot L}{A \cdot E}$$

FBD



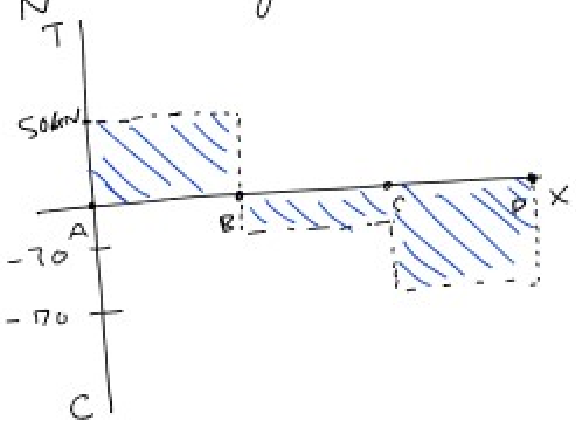
Equil

$\overset{+}{\rightarrow} \sum F_x$: $N_{CD} = 70$ kN (C)

$\pm \sum F_x$ $N_{BC} = 30$ kN (C)

$\pm \sum F_x$: $N_{AB} = -50$ kN (T)

Normal Force Diagram



a) $\delta_D = ?$

$$\delta_D = \sum \frac{P \cdot L}{A E} = \frac{N_{CD} L_{CD}}{A E} + \frac{N_{BC} L_{BC}}{A E} + \frac{N_{AB} L_{AB}}{A E}$$

Tables → A-36 → $E = 200\ GPa$

$$A = \pi r^2 = \frac{\pi}{4}(d)^2 = \pi (25\ mm)^2$$

$$\delta_D = \frac{[-70(10^3)\ N][1.5(10^3)\ mm]}{\pi (25\ mm)^2\ [200(10^3)\ N/mm^2]}$$

$$+ \frac{[-30(10^3)\ N][1\ (10^3)\ mm]}{''\ [\ ''\ ]}$$

$$+ \frac{[50(10^3)\ N][2(10^3)\ mm]}{''\ [\ ''\ ]}$$

$$\boxed{\delta_D = -89.1(10^{-3})\ mm}$$

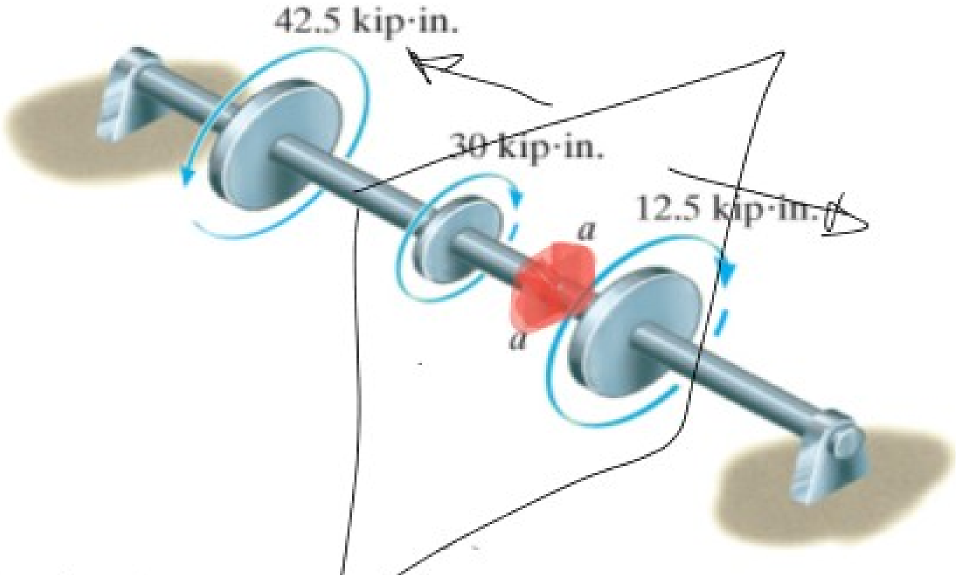
b) $\delta_{BC} = ?$

$$\boxed{\delta_{BC} = -76.4(10^{-3})\ mm}$$

$$\sigma = \frac{F}{A}$$

$$F = \sigma \cdot A$$

$$\Sigma F = 0$$

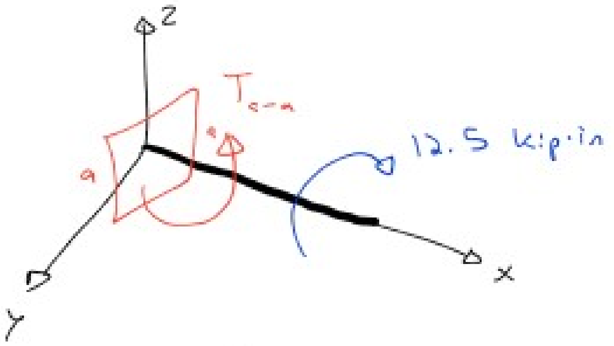


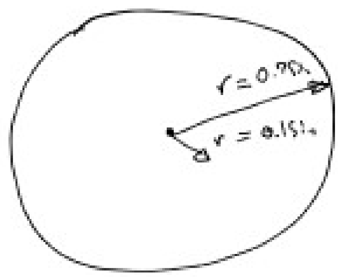
K & UKs

K: T_1, T_2, T_3 $d = 1.5\ in$,

UK: $\tau_a = ?$ $\tau_b = ?$ $\rightarrow \tau = \frac{T \cdot \rho}{J} \rightarrow T_{a-a} = ?, J = ?$

FBD Method Sections

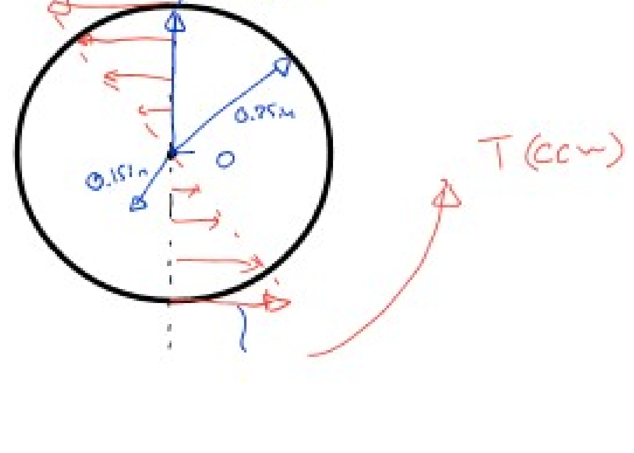




$$\rightarrow \Sigma M_x : -12.5 + T = 0$$

$$T = 12.5\ kip \cdot in\ ccw$$

Stress Distribution Diagram



Torsional Shear Stress

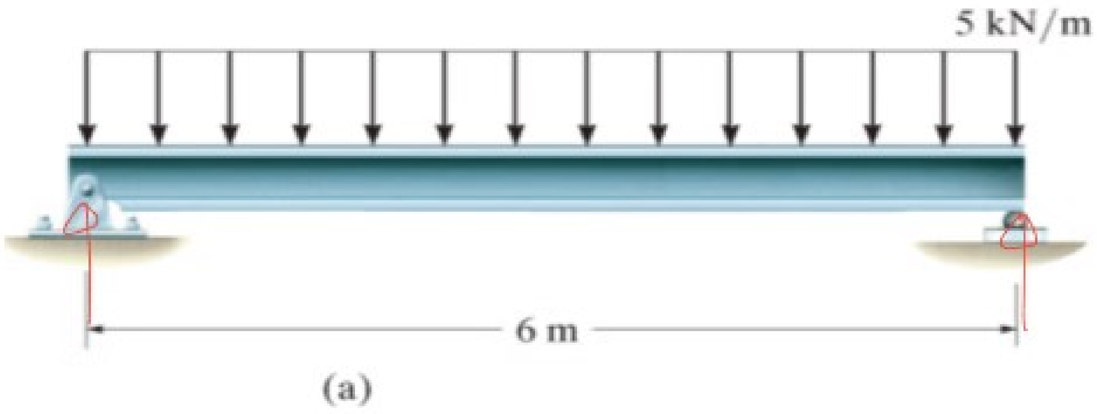
$$\tau = \frac{T \cdot \rho}{J} = \begin{cases} \tau_a = \dfrac{T_{e\text{-}a} \cdot \rho_a}{J} \\ \tau_b = \dfrac{T_{e\text{-}a} \cdot \rho_b}{J} \end{cases}$$

$$J = \frac{\pi}{2} r^4 = \frac{\pi}{2} (0.75 \text{ in})^4 = 0.497 \text{ in}^4$$

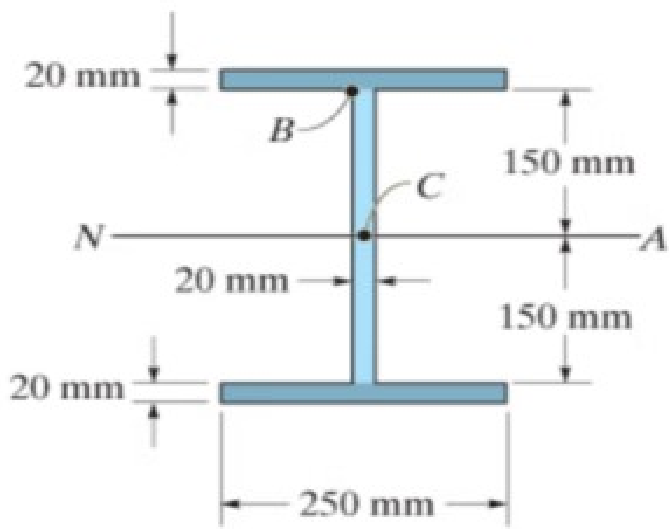
$$\tau_a = \frac{(12.5 \text{ kip} \cdot \text{in})(0.75 \text{ in})}{''} = 18.9 \text{ ksi}$$

$$\tau_b = \frac{(\quad '' \quad)(0.15 \text{ in})}{''} = 3.77 \text{ ksi}$$

Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location. Also what is the stress at point B?



(a)



K & UKs

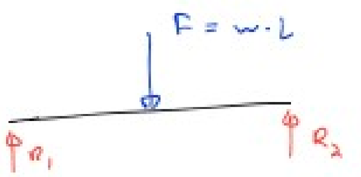
K: Load, Supports, Dimensions

$M_{max} = ?$

UKs: $R_1, R_2,$ $\sigma_{max} = ? = \frac{M(x) \cdot c}{I}$

M(x) Diagram → V(x) Diagram

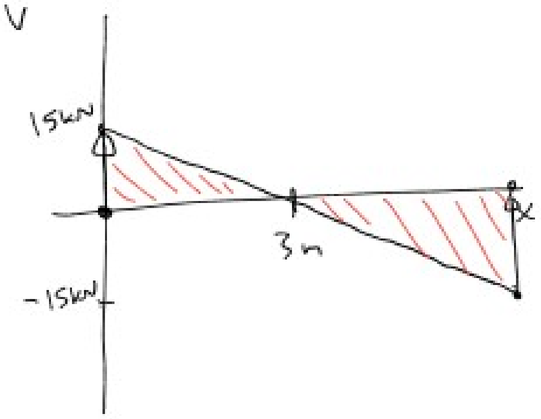
FBD



Equil

$R_1 = R_2 = 15\text{ kN}$

Shear Force Diagram



$$\Delta V = F_o$$

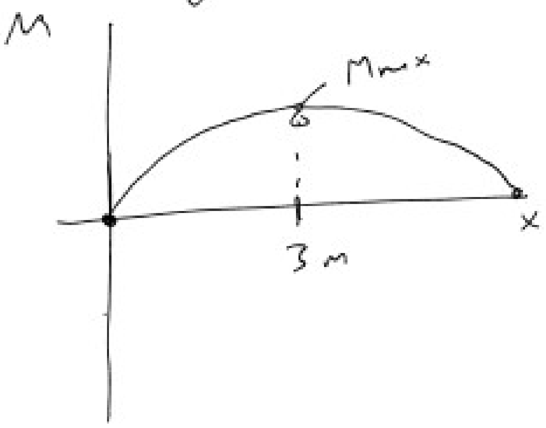
$$\frac{dV}{dx} = w(x)$$

$$V = \int w(x)\,dx$$

$$V(x) = \int -5\,dx + c = -5x + c$$

$$V(x) = -5x + 15$$

Bending Moment Diagram



$$\frac{dM}{dx} = V(x)$$

$$M = \int V(x)\,dx + C$$

$$M = \int -5x + 15\,dx + C$$

$$M(x) = \frac{-5x^2}{2} + 15x + c$$

$$M(0) = 0 + 0 + c$$

$$c = 0$$

$$M_{max} \Rightarrow \frac{dM}{dx} = V = 0$$

$x = 3m$ $\quad M(3) = \boxed{M_{max}} = \frac{-5(3)^2}{2} + 15(3) = \boxed{22.5\ kN \cdot m}$

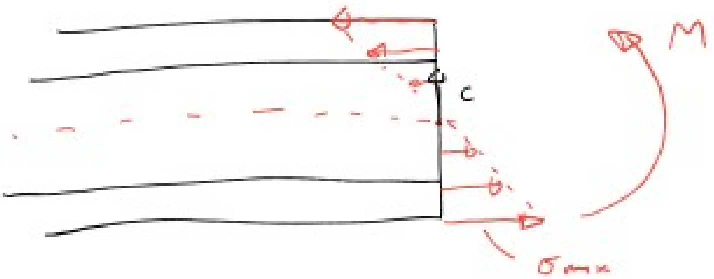
Abs Maximum Bending Stress, σ_{max}

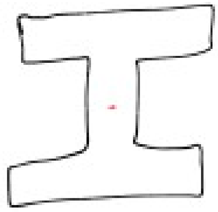
$$\sigma_{max} = \frac{M_{max} \cdot c}{I}$$

We search the tables to find I

$$\boxed{I = 301.3\ E{-6}\ m^4}$$

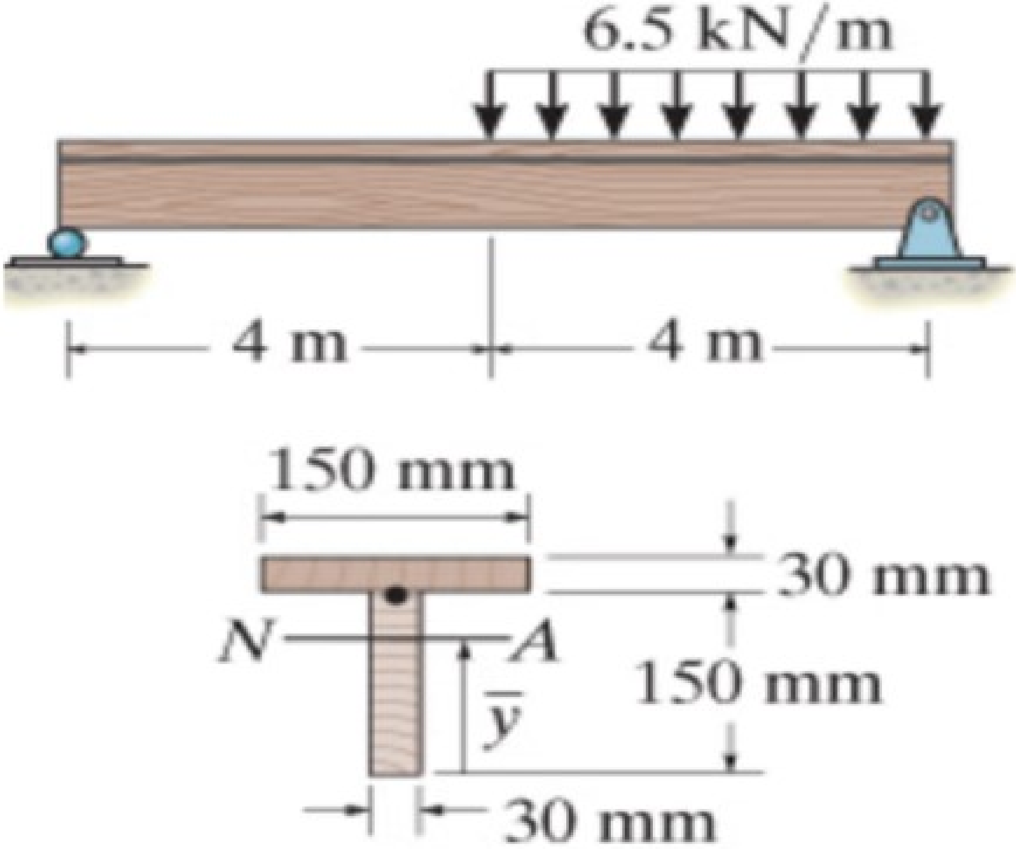
Stress Distribution





$$\sigma_{max} = \frac{M_{max} \cdot c}{I} = 12.7 \text{ MPa}$$

The Beam is made of two boards. Determine the maximum shear stress in the glue necessary to hold the boards together alog the seam where they are joined.



K & Uks

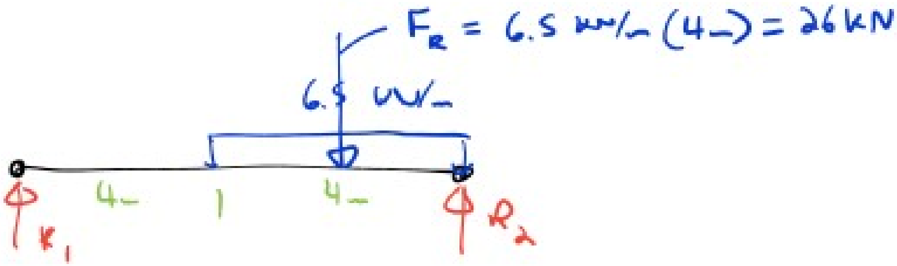
type of support, dimensions, $w = 6.5$ kN/m

$\bar{y} = ?$, $\tau_{max,glue} = ?$, $V(x) = ?$ ✓, $Q = ?$ ✓, $I = ?$ ✓, $t = ?$ ✓

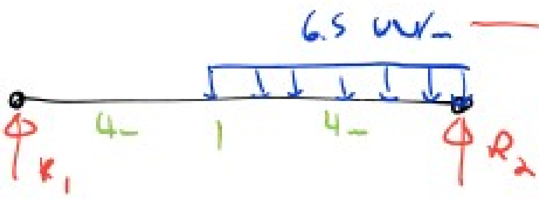
Reactions = ? ✓

$$\tau_{max,glue} = \frac{V_{max}\, Q}{I \cdot t}$$

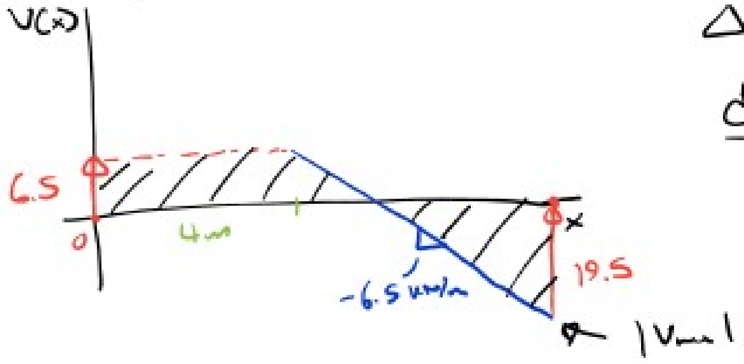
FBD



ΣF & ΣM → $R_1 = 6.5$ kN, $R_2 = 19.5$ kN



Shear Force Diagram

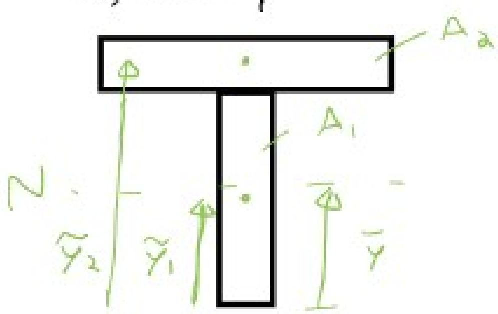


$$\Delta V = F_c$$

$$\frac{dV}{dx} = w(x)$$

Find, I the 2nd Moment of Inertia

a) Find $\bar{y}$



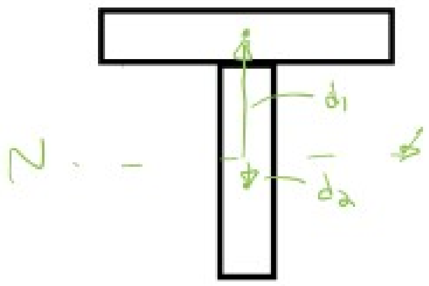
Centroid, Neutral Axis — $\bar{y}$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

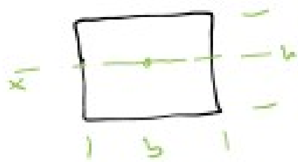
$$\bar{y} = \frac{\tilde{y}_1 A_1 + \tilde{y}_2 A_2}{A_1 + A_2}$$

$$\bar{y} = 0.12\ m$$

b) Find, I



The I of a Single Rectangle



$$I_x = \frac{bh^3}{12}$$

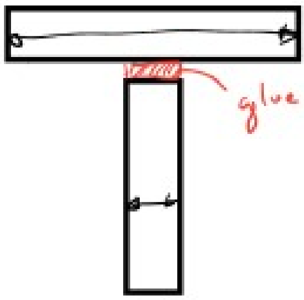
Parallel Axis theorem

$$I = \Sigma(I_x + Ad^2)$$

$$I = (I_{x1} + A_1 d_1^2) + (I_{x2} + A_2 d_2^2)$$

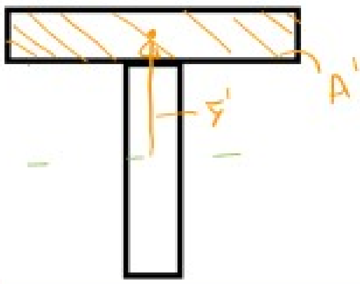
$$I = 27E\text{-}6\ m^4$$

Find, t = ?



$t = 0.03\ m$

Find, Q = ?



$$\tau = \frac{VQ}{It}$$

the top of the board is held in place by the bottom. So Q should be taken from the top because it will produce a larger, τ!

$$Q = \bar{y}'A'$$

$\bar{y}'$: centroid of A' from the neutral axis

A': Surface Area of Interest

$Q = 0.2025E\text{-}3\ m^3$

Solve for $\tau_{max,glue}$

$$\tau_{max,glue} = \frac{V_{max}Q}{It} \rightarrow \tau_{max,glue} = 4.88\ MPa$$