1.22	ALLOY STEEL FIG. 1.3 & 1.4	STRUCTURAL STEEL FIG. 1.5
YIELD POINT	N/A	265 MPa
YIELD STRENGTH	450 MPa	N/A
UPPER YIELD POINT	N/A	280 MPa
LOWER YIELD POINT	N/A	265 MPa
MODULUS OF RESILIENCE	N/A	0.1855 MPa
ULTIMATE TENSILE STRENGTH	715 MPa	470 MPa
STRAIN AT FRACTURE	0.23	0.26
PER CENT ELONGATION	23%	26%

1.24 ASSUME: 1. PLANE SECTIONS NORMAL TO THE AXIS OF THE BAR REMAIN PLANE UNDER APPLICATION OF THE LOAD.
2. HOOKE'S LAW APPLIES.

EQUILIBRIUM:

$$P = \sigma A \quad (a)$$

HOOKE'S LAW:

$$\sigma = E\varepsilon \quad (c)$$

CONTINUITY:

$$\Delta L = \int_0^L \varepsilon \, dx \quad (b)$$

GEOMETRY

$$A(x) = b(d_0 - x/L(d_0 - d_1)) \quad (d)$$

SUB (d) INTO (a)

$$\sigma = P/[b(d_0 - x/L(d_0 - d_1))] \quad (e)$$

SUB. (c) & (e) INTO (b):

$$\Delta L = \int_0^L \frac{P}{Eb}\left[d_0 - x/L(d_0 - d_1)\right]^{-1} dx$$

$$\Delta L = \frac{PL}{Eb}\int_0^L \frac{dx}{d_0 L - x(d_0 - d_1)} = \frac{PL}{Eb}\left(\frac{1}{d_0 - d_1}\right)\ln\frac{d_0}{d_1}$$

1.25

RODS: $A_R = 4\left(\frac{\pi}{4}15^2\right) = 706.9\ mm^2$

PIPE: $A_p = \frac{\pi}{4}(100^2 - 90^2) = 1492\ mm^2$

AFTER ASSEMBLY: $T_R = C_p = 4(65) = 260\ kN$

$$\sigma_R = \frac{4(65000)}{706.9} = 367.8\ MPa$$

$$\sigma_p = \frac{-4(65000)}{1492} = -174.3\ MPa$$

AFTER PRESSURE IS APPLIED:

EQUILIBRIUM:

$$p\left(\frac{\pi}{4}90^2\right) = \Delta T_R + \Delta C_p \quad (a)$$

COMPATIBILITY:

$$\Delta L_R = \Delta L_p: \quad \frac{\Delta T_R L}{A_R E} = \frac{\Delta C_p L}{A_p E} \quad (b)$$

LEAKAGE REQUIREMENT:

$$\Delta C_p = 260\ kN \quad (c)$$

SUB (c) INTO (b):

$$\Delta T_R = 123.2\ kN \quad (d)$$

SUB (c) & (d) INTO (a)

$$\underline{\underline{p = 60.23\ MPa}}$$

$$\Delta\sigma_R = \frac{123200}{706.9} = 174.3\ MPa$$

$$\underline{\underline{\sigma_{R(FINAL)} = 542.1\ MPa}}$$

1.26

(a) Figure a shows the bars subjected to force P. Figure b shows the free-body diagrams of the steel bar, the aluminum bar, and point A. By the free-body diagram of point A,

$$P = P_s + P_a \qquad (a)$$

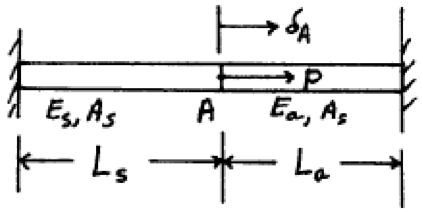

Figure a

By Eq. (1.2) and Figs. a and b,

$$\delta_a = \frac{P_s L_s}{E_s A_s} = \frac{P_a L_a}{E_a A_a} \qquad (b)$$

By Eqs. (a) and (b),

$$P_s = \frac{P_a E_s A_s L_a}{E_a A_a L_s} = \frac{(P - P_s)(E_s A_s L_a)}{E_a A_a L_s}$$

Solving this equation for P_s, we find

$$P_s = \frac{P E_s A_s L_a}{E_s A_s L_a + E_a A_a L_s}$$. Hence, by Eq. (1.1),

the stress in the steel bar is

$$\sigma_s = \frac{P_s}{A_s} = \frac{P}{A_s}\left(\frac{E_s A_s L_a}{E_s A_s L_a + E_a A_a L_s}\right)$$

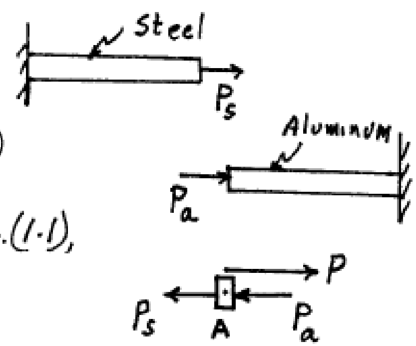

Figure b

Similarly, by Eqs. (a) and (b),

$$P_a = \frac{P_s E_a A_a L_s}{E_s A_s L_a} = \frac{(P - P_a)(E_a A_a L_s)}{E_s A_s L_a}$$

or

$$P_a = \frac{P E_a A_a L_s}{E_s A_s L_a + E_a A_a L_s}$$

Hence, the stress in the aluminum bar is

$$\sigma_a = \frac{P}{A_a}\left(\frac{E_a A_a L_s}{E_s A_s L_a + E_a A_a L_s}\right)$$

(cont.)

1.26 cont.

(b) When the left wall is displaced to the right by an amount δ, the point A is displaced to the right by an amount δ_A (Fig. c).

By Eq. (1.2) and Fig. c,

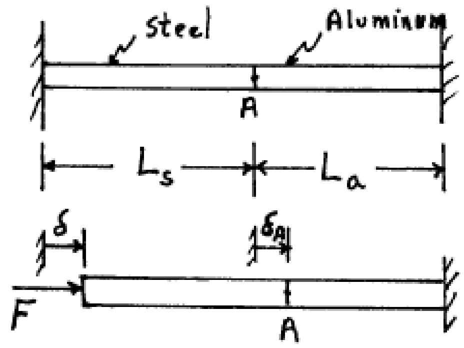

Figure C

$$\delta_A = \frac{FL_a}{E_a A_a} \quad (a)$$

and

$$\delta - \delta_A = \frac{FL_s}{E_s A_s} \quad (b)$$

By Eqs. (a) and (b),

$$\delta = \delta_A + \frac{FL_s}{E_s A_s} = F\left(\frac{L_a}{E_a A_a} + \frac{L_s}{E_s A_s}\right)$$

or

$$F = \frac{\delta E_a A_a E_s A_s}{E_s A_s L_a + E_a A_a L_s}$$

Hence, by Eq. (1.1), the stress in the steel bar is

$$\sigma_s = \frac{F}{A_s} = \frac{\delta E_a A_a E_s}{E_s A_s L_a + E_a A_a L_s}$$

and the stress in the aluminum bar is

$$\sigma_a = \frac{F}{A_a} = \frac{\delta E_a E_s A_s}{E_s A_s L_a + E_a A_a L_s}$$

1.31

The stress-strain curve is shown in Fig. a.

By Fig. a, the ultimate strength is $\sigma_u \approx 665$ MPa

Since each rectangular box under the stress-strain curve represents $(100)(0.05) = 5\ MN{\cdot}m/m^3$ of energy and we estimate that there are approximately 29.5 boxes under the stress-strain curve, we find

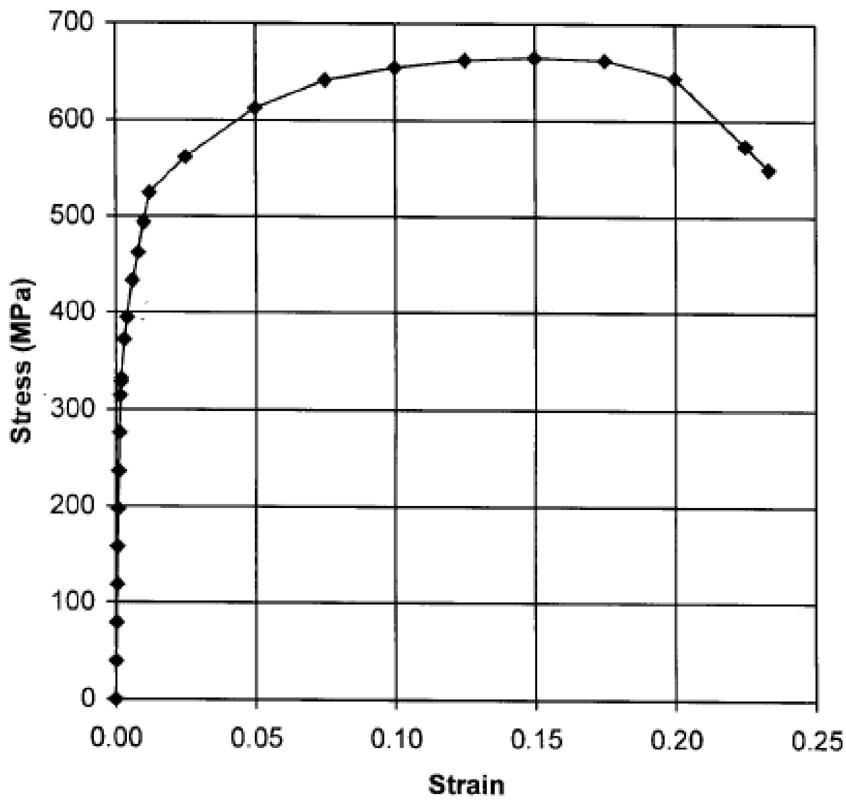

Figure a

that the modulus of toughness is

$$U_F = (5)(29.5) = 147.5\ MN{\cdot}m/m^3.$$

By numerical integration,

$$U_F = 144.2\ MN{\cdot}m/m^3$$

1.32

Extending the straight line portion of the stress-strain curve (Fig. a), we see that the slope of the line is

$$E = \frac{400}{.002} = 200 \text{ GPa}.$$

also, by Fig. a,

$$\sigma_{YS} = 395 \text{ MPa} \quad \text{and} \quad \sigma_{PL} = 315 \text{ MPa}$$

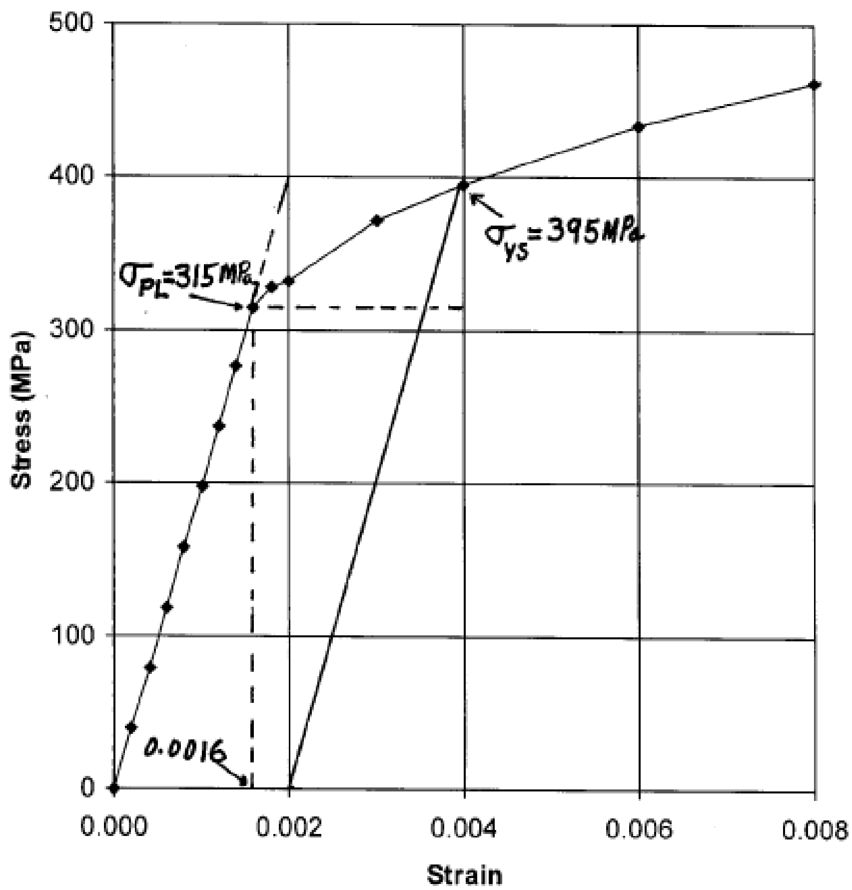

Figure a

The area under the stress-strain curve to the yield strength is approximately (see Fig. a)

$$\text{Area} = \frac{1}{2}(315)(0.0016) + (315)(0.004 - 0.016) + \frac{1}{2}(395 - 315)(0.004 - 0.0016)$$

or,

$$\text{Modulus of Resilience} = 1.104 \ \text{MN}\cdot\text{m/m}^3$$