Lecture 2

Example 1

$$\underline{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \underline{e}_2 = \begin{bmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} \qquad \underline{e}_3 = \begin{bmatrix} 0 \\ -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

Prove $\underline{e}_i \cdot \underline{e}_j = \delta_{ij}$ where $\delta_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$i = 1, 2, 3$

$j = 1, 2, 3$

$$\underline{e}_1 \cdot \underline{e}_2 = \delta_{12} = (1)(0) + (0)\left(\frac{\sqrt{2}}{2}\right) + (0)\left(\frac{\sqrt{2}}{2}\right) = 0$$

$$\underline{e}_2 \cdot \underline{e}_1 = \underline{e}_1 \cdot \underline{e}_2 = 0$$

$$\underline{e}_2 \cdot \underline{e}_3 = (0)(0) + \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = 0$$

$$\underline{e}_3 \cdot \underline{e}_2 = 0$$

$$\underline{e}_3 \cdot \underline{e}_1 = (0)(1) + \left(\frac{-\sqrt{2}}{2}\right)(0) + \left(\frac{\sqrt{2}}{2}\right)(0) = 0$$

$$\underline{e}_1 \cdot \underline{e}_3 = 0$$

$$\underline{e}_1 \cdot \underline{e}_1 = (1)(1) + 0 + 0 = 1$$

$$\underline{e}_2 \cdot \underline{e}_2 = (0)(0) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = 1$$

$$\underline{e}_3 \cdot \underline{e}_3 = (0)(0) + \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = 1$$

thus,

$$\underline{e}_i \cdot \underline{e}_j = \delta_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

Example 2

Prove $\underline{u}\cdot\hat{\imath}$, $\underline{u}\cdot\hat{\jmath}$, $\underline{u}\cdot\hat{k}$

equal u_x , u_y , u_z

$$\underline{u} = u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k}$$

$$\underline{u}\cdot\hat{\imath} = (u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k})\cdot(\hat{\imath}) \quad \times\hat{\jmath} \quad \times\hat{k}$$

$$= u_x\overset{1}{\cancel{\hat{\imath}\cdot\hat{\imath}}} + 0 + 0$$

$\underline{u}\cdot\hat{\imath} = u_x$ ✓

$\underline{u}\cdot\hat{\jmath} = u_y$ ✓

$\underline{u}\cdot\hat{k} = u_z$ ✓

Example 3

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

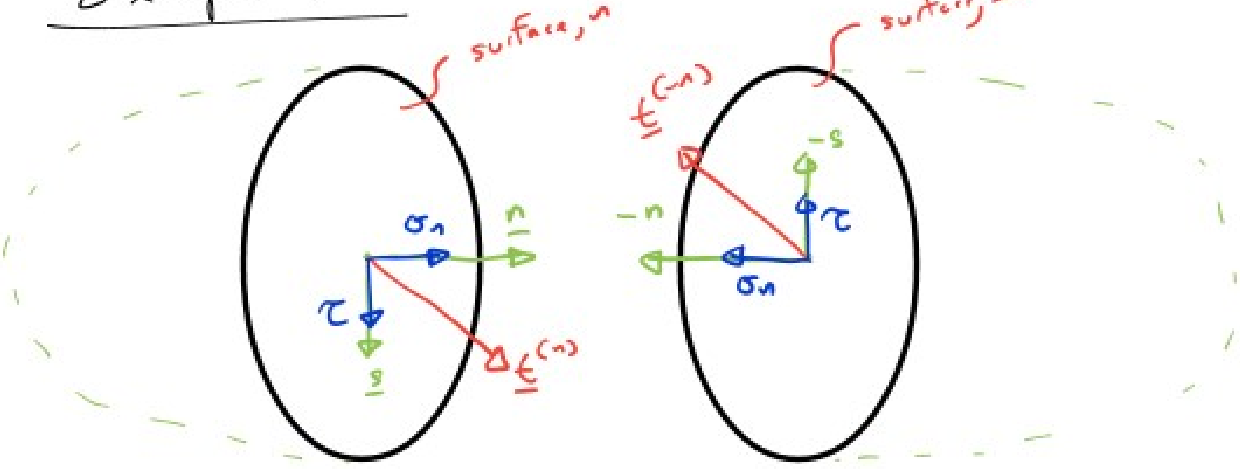
$$\underline{\underline{A}} \cdot \underline{b}$$

$$\underline{\underline{A}} \cdot \underline{b} = \underset{3 \times \cancel{3}}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}} \underset{\cancel{3} \times 1}{\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}}$$

$$\underline{\underline{A}} \cdot \underline{b} = \begin{bmatrix} (1)(0) + (2)(-1) + (3)(1) \\ (4)(0) + (5)(-1) + (6)(1) \\ (7)(0) + (8)(-1) + (9)(1) \end{bmatrix}$$

$$3 \times 1$$

Example 4



For Equil. & Using the Method of Sections

$$\Sigma \underline{F} = 0 \qquad \underline{t}^{(n)} = \frac{\Delta \underline{F}}{\Delta A} = \frac{d\underline{F}}{dA}$$

$$\Sigma \underline{F} : \quad \underline{t}^{(n)} \cdot dA - \underline{t}^{(-n)} \cdot dA = 0$$

$$\Sigma F_n = 0$$

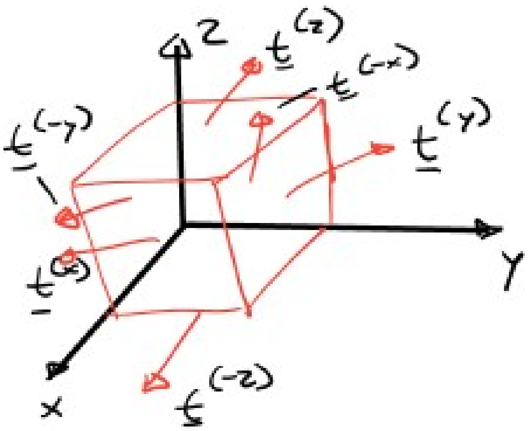
$$\Sigma F_s = 0$$

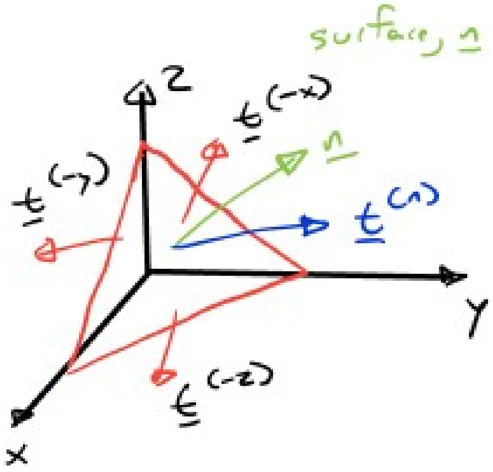
$$\Sigma \underline{F} : (\sigma_n \underline{n} + \tau \underline{s}) dA - (\sigma_n \underline{n} + \tau \underline{s}) dA = 0$$

$$\overset{+}{\rightarrow} \Sigma F_n : \quad \sigma_n dA - \sigma_n dA = 0$$

$$+\!\downarrow \Sigma F_s : \quad \tau \, dA - \tau \, dA = 0$$

Derive

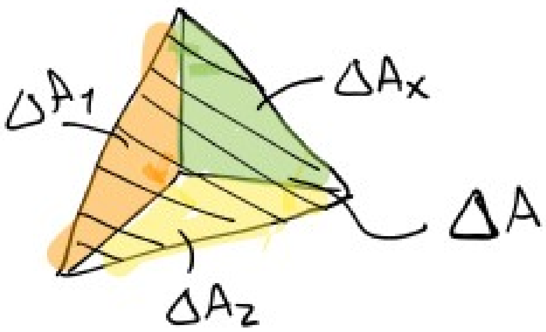




Static Equilibrium Method of Sections

$$\sum \underline{F} = 0 \qquad \underline{t} = \frac{\Delta F}{\Delta A}$$

$$\sum \underline{F}: \ \underline{t}^{(n)}\Delta A + \underline{t}^{(-x)}\Delta A_x + \underline{t}^{(-y)}\Delta A_y + \underline{t}^{(-z)}\Delta A_z = 0$$



$$\frac{\Delta A_x}{\Delta A} = n_x$$

$$\frac{\Delta A_y}{\Delta A} = n_y$$

$$\frac{\Delta A_z}{\Delta A} = n_z$$

$$\underline{n} = n_x \hat{\imath} + n_y \hat{\jmath} + n_z \hat{k}$$

$$\sqrt{n_x^2 + n_y^2 + n_z^2} = 1$$

Lets divide $\sum \underline{F}$ by ΔA

$$\sum \underline{F}: \underline{t}^{(n)} + \underline{t}^{(-x)} n_x + t^{(-y)} n_y + t^{(-z)} n_z = 0$$

Remember,

$$\underline{t}^{(-x)} = -\underline{t}^{(x)}, \quad \underline{t}^{(-y)} = -\underline{t}^{(y)}, \quad \underline{t}^{(-z)} = -\underline{t}^{(z)}$$

$$\boxed{\underline{t}^{(n)} = \underline{t}^{(x)} n_x + \underline{t}^{(y)} n_y + \underline{t}^{(z)} n_z}$$

$$\underline{t}^{(n)} \longrightarrow \hat{\imath}, \hat{\jmath}, \hat{k}$$

$$\underline{t}^{(n)} = t_x^{(n)} \hat{\imath} + t_y^{(n)} \hat{\jmath} + t_z^{(n)} \hat{k}$$

$$t_x^{(n)} = \underline{t}^{(n)} \cdot \hat{\imath}$$

$$t_x^{(n)} = \left(\underline{t}^{(x)} n_x + \underline{t}^{(y)} n_y + t^{(z)} n_z \right) \cdot \hat{\imath}$$

$$t_x^{(n)} = \underbrace{\left(\underline{t}^{(x)} \cdot \hat{\imath} \right)}_{\text{surface} \quad \text{direct}} n_x + \left(\underline{t}^{(y)} \cdot \hat{\imath} \right) n_y + \left(\underline{t}^{(z)} \cdot \hat{\imath} \right) n_z$$

① $t_x^{(n)} = \sigma_{xx} n_x + \sigma_{yx} n_y + \sigma_{zx} n_z$

② $t_y^{(n)} = \sigma_{xy} n_x + \sigma_{yy} n_y + \sigma_{yz} n_z$

③ $t_z^{(n)} = \sigma_{xz} n_x + \sigma_{yz} n_y + \sigma_{zz} n_z$

$$\underline{t}^{(n)} \qquad \underline{\underline{\sigma}} \qquad \underline{n}$$

$$\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}^{(n)} = \begin{bmatrix} \sigma_{xx} & \sigma_{yy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

$$\underline{t}^{(n)} = \underline{\underline{\sigma}} \cdot \underline{n}$$

Example 5

$$\underline{\underline{\sigma}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & \sigma_{yy} & 2 \\ 1 & 2 & 0 \end{bmatrix} \qquad \underline{t} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{n} = n_1 \underline{e}_1 + \underline{n}_2 \underline{e}_2 + \underline{n}_3 \underline{e}_3$$

Unks $\sigma_{yy}, n_1, n_2, n_3$

$$\underline{t} = \underline{\underline{\sigma}} \cdot \underline{n}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & \sigma_{yy} & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

(1) $n_1 + n_3 = 0$

(2) $\sigma_{yy} n_2 + 2 n_3 = 0$

(3) $n_1 + 2n_2 = 0$

$\underline{n}$ is a unit direction vector

(4) $\sqrt{n_1^2 + n_2^2 + n_3^2} = 1$

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take (1) solve for n_1

$n_1 = -n_3$ (I)

take (2) solve for σ_{yy}

$\sigma_{yy} = \dfrac{-2n_3}{n_2}$ (II)

take (3) solve for n_2

$n_2 = \dfrac{-n_1}{2}$ (III)

$$\sigma_{yy} = \frac{-2n_3}{n_2} = \frac{-2n_3}{-\frac{n_1}{2}} = \frac{-2n_3}{\frac{n_3}{2}}$$

$$\sigma_{yy} = 4$$

Using I & III & 4

$$\sqrt{n_1^2 + n_2^2 + n_3^2} = \sqrt{n_1^2 + \left(-\frac{n_1}{2}\right)^2 + (-n_1)^2} = 1$$

$$\sqrt{2.25}\; n_1 = 1$$

$$n_1 = 0.66\bar{6} = \frac{2}{3}$$

Continue to find

n_2 & n_3

Example 6

$$\sigma = \begin{bmatrix} 10 & 3 & 5 \\ 3 & 1 & 2 \\ 5 & 2 & 4 \end{bmatrix} \text{ ksi}$$

$$n_x = \frac{\sqrt{3}}{2}, \quad n_y = \frac{\sqrt{2}}{4}, \quad n_z = -\frac{\sqrt{2}}{4}$$

Find σ_n, τ, $\underline{n}$, $\underline{s}$, $|\underline{t}|$, θ $\quad \underline{n} : \theta \; t_n$

$$\underline{t} = \underline{\underline{S}} \cdot \underline{n}$$

$$\sigma_n = \underline{t} \cdot \underline{n}$$

$$\tau = \left[\underline{t} \cdot \underline{t} - \sigma_n^2 \right]^{1/2}$$

$\underline{n}$ – given

$$\underline{t} = \sigma_n \underline{n} + \tau \underline{s}$$

$$\underline{s} = \frac{\underline{t} - \sigma_n \underline{n}}{\tau}$$

$$\underline{t} \cdot \underline{n} = |\underline{t}| \, |\underline{n}| \cos \theta$$

Example 5

Knowns | **Unks**

$$\underline{\underline{\sigma}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & \sigma_{yy} & 2 \\ 1 & 2 & 0 \end{bmatrix}, \quad \underline{t}^{(n)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\sigma_{yy} = ?$

$$\underline{n} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{matrix} ? \\ ? \\ ? \end{matrix}$$

4 Unks.
$\downarrow$
4 Eqn.

Traction on a Arbitrary Plane, $\underline{t}^{(n)}$

$$\underline{t}^{(n)} = \underline{\underline{\sigma}} \cdot \underline{n}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & \sigma_{yy} & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

① $n_1 + n_3 = 0$

② $\sigma_{yy} n_2 + 2 n_3 = 0$

③ $n_1 + 2 n_2 = 0$

$\underline{n}$ is unit direction vector such that

④ $\sqrt{n_1^2 + n_2^2 + n_3^2} = 1$

take ① solve n_1

$$n_1 = -n_3 \quad (\mathrm{I})$$

take ② solve σ_{yy}

$$\sigma_{yy} = \frac{-2 n_3}{n_2} \quad (\mathrm{II})$$

take ③ solve for n_2

$$n_2 = -\frac{n_1}{2} \quad (\mathrm{III})$$

take (II)

$$\sigma_{yy} = \frac{-2 n_3}{n_2} = \frac{-2 n_3}{-\frac{n_1}{2}} = \frac{2(-\cancel{n_1})}{\frac{\cancel{n_1}}{2}} = -4$$

$$\boxed{\sigma_{yy} = -4}$$

Example 6 (2-4 Solecki)

$$\underline{\underline{\sigma}} = \begin{bmatrix} 10 & 3 & 5 \\ 3 & 1 & 2 \\ 5 & 2 & 4 \end{bmatrix} \text{ksi}$$

$n_x = \frac{\sqrt{3}}{2}$

$n_y = \frac{\sqrt{2}}{4}$

$n_z = -\frac{\sqrt{2}}{4}$

$$\underline{n} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

① Find the σ_n & τ the normal & shear component of the traction on plane n.

② Find the angle or direction cosines of σ_n & τ with respect to the $\underline{t}^{(n)}$

③ Find the Mag of traction vector, $\underline{t}^{(n)}$ & its angle with the normal, $\underline{n}$!

Equations we Needed for ①

$$\underline{t}^{(n)} = \underline{\underline{\sigma}} \cdot \underline{n}$$

$$\sigma_n = \underline{t}^{(n)} \cdot \underline{n}$$

$$\tau = \left[\underline{t}^{(n)} \cdot \underline{t}^{(n)} - \sigma_n^2 \right]^{1/2}$$

Part ①

$$\underline{t}^{(n)} = \underline{\underline{\sigma}} \cdot \underline{n} = \begin{bmatrix} 10 & 3 & 5 \\ 3 & 1 & 2 \\ 5 & 2 & 4 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}}{4} \end{bmatrix} =$$

$$\underline{t}^{(n)} = \begin{bmatrix} 10\left(\frac{\sqrt{3}}{2}\right) + 3\left(\frac{\sqrt{2}}{4}\right) - 5\left(\frac{\sqrt{2}}{4}\right) \\ 3\left(\frac{\sqrt{3}}{2}\right) + 1\left(\frac{\sqrt{2}}{4}\right) - 2\left(\frac{\sqrt{2}}{4}\right) \\ 5\left(\frac{\sqrt{3}}{2}\right) + 2\left(\frac{\sqrt{2}}{4}\right) - 4\left(\frac{\sqrt{2}}{4}\right) \end{bmatrix}$$

$$\sigma_n = \underline{t}^{(n)} \cdot \underline{n} = (t_x \hat{i} + t_y \hat{j} + t_z \hat{k}) \cdot (n_x \hat{i} + n_y \hat{j} + n_z \hat{k})$$

$$= t_x n_x + t_y n_y + t_z n_z$$

$$\boxed{\sigma_n = 6.461 \text{ ksi}}$$

$$\tau = \left[\underline{t}^{(n)} \cdot \underline{t}^{(n)} - \sigma_n^2 \right]^{1/2}$$

$$\tau = \left[(t_x t_x + t_y t_y + t_z t_z) - \sigma_n^2 \right]^{1/2}$$

$$\boxed{\tau = 6.33 \text{ ksi}}$$

Equations Needed for Part (2)

$$\underline{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \quad \& \quad \underline{s} = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}$$

Given Done!

$$\underline{t}^{(n)} = \sigma_n \underline{n} + \tau \underline{s}$$

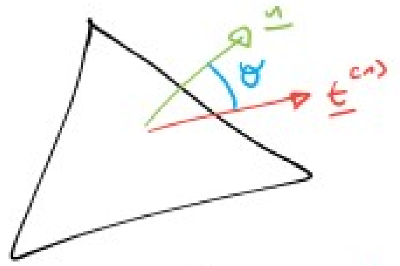
$$\underline{s} = \frac{\underline{t}^{(n)} - \sigma_n \underline{n}}{\tau} = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix} = \begin{bmatrix} \left(t_x - \sigma_n \frac{\sqrt{3}}{2}\right) / 6.37\text{ ksi} \\ \left(t_y - \sigma_n \frac{\sqrt{2}}{4}\right) / 6.37\text{ ksi} \\ \left(t_z + \sigma_n \frac{\sqrt{2}}{4}\right) / 6.37\text{ ksi} \end{bmatrix}$$

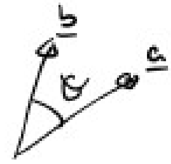
$s_x = 0.379$, $s_y = -0.011$, $s_z = 0.925$

verify

$$\sqrt{s_x^2 + s_y^2 + s_z^2} = 1$$

Equations for part ③





$$\underline{a} \cdot \underline{b} = |a||b| \cos\theta$$

$$\sigma_n = \underline{t}^{(n)} \cdot \underline{n} = |\underline{t}^{(n)}| \cos\theta \qquad (1)$$

$$|\underline{t}^{(n)}| = \sqrt{t_x^2 + t_y^2 + t_z^2}$$

$$|\underline{t}^{(n)}| = 9.023 \text{ ksi}$$

solve for θ

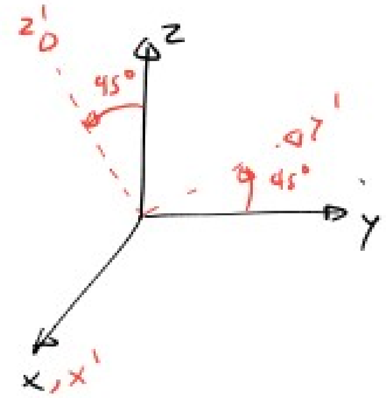
$$\theta = \cos^{-1}\left(\frac{\sigma_n}{|\underline{t}^{(n)}|}\right)$$

$$\boxed{\theta = 40.6^\circ}$$

Example 7

Given a Cauchy stress tensor that is in a xyz coordinate system. The coord. sys is rotated by 45° CCW about the +x axis. Find the transformed Cauchy stress Tensor, $\underline{\underline{\sigma}}'$.

$$\underline{\underline{\sigma}} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$



Find our $\underline{n}_{x'}$, $\underline{n}_{y'}$, $\underline{n}_{z'}$ vectors

$\underline{n}_{x'} \rightarrow n_{x'x} = 1$, $n_{x'y} = 0$, $n_{x'z} = 0$

$\underline{n}_{y'} \rightarrow n_{y'x} = 0$, $n_{y'y} = \cos\theta$, $n_{y'z} = \sin\theta$

$\underline{n}_{z'} \rightarrow n_{z'x} = 0$, $n_{z'y} = -\sin\theta$, $n_{z'z} = \cos\theta$

$$\underline{\underline{N}} = \left[\underline{n}_{x'} \,\vdots\, \underline{n}_{y'} \,\vdots\, \underline{n}_{z'}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\underline{\underline{N}}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

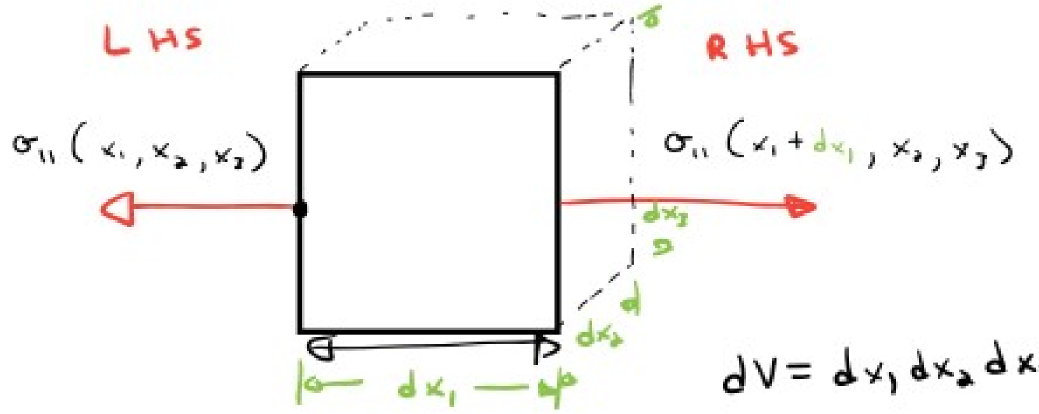
$$\underline{\underline{\sigma}}' = \underline{\underline{N}}^T \cdot \left(\underline{\underline{\sigma}} \cdot \underline{\underline{N}}\right) = \begin{bmatrix} 0 & 2.121 & 0.707 \\ 2.121 & 6 & 1 \\ 0.707 & 1 & 0 \end{bmatrix}$$

Derive

Equilibrium

Lets consider a 3D element subject to

Normal Stress in the x_1 direction.



$dV = dx_1 \, dx_2 \, dx_3$

Look at the RHS. According to Taylor's theorem,

$$\underbrace{\sigma_{11}(x_1 + dx_1, x_2, x_3)}_{\text{RHS}} = \underbrace{\sigma_{11}(x_1, x_2, x_3)}_{\text{LHS}} + \underbrace{dx_1 \frac{d\sigma_{11}}{dx_1}(x_1, x_2, x_3)}_{\text{Variation across the element}}$$

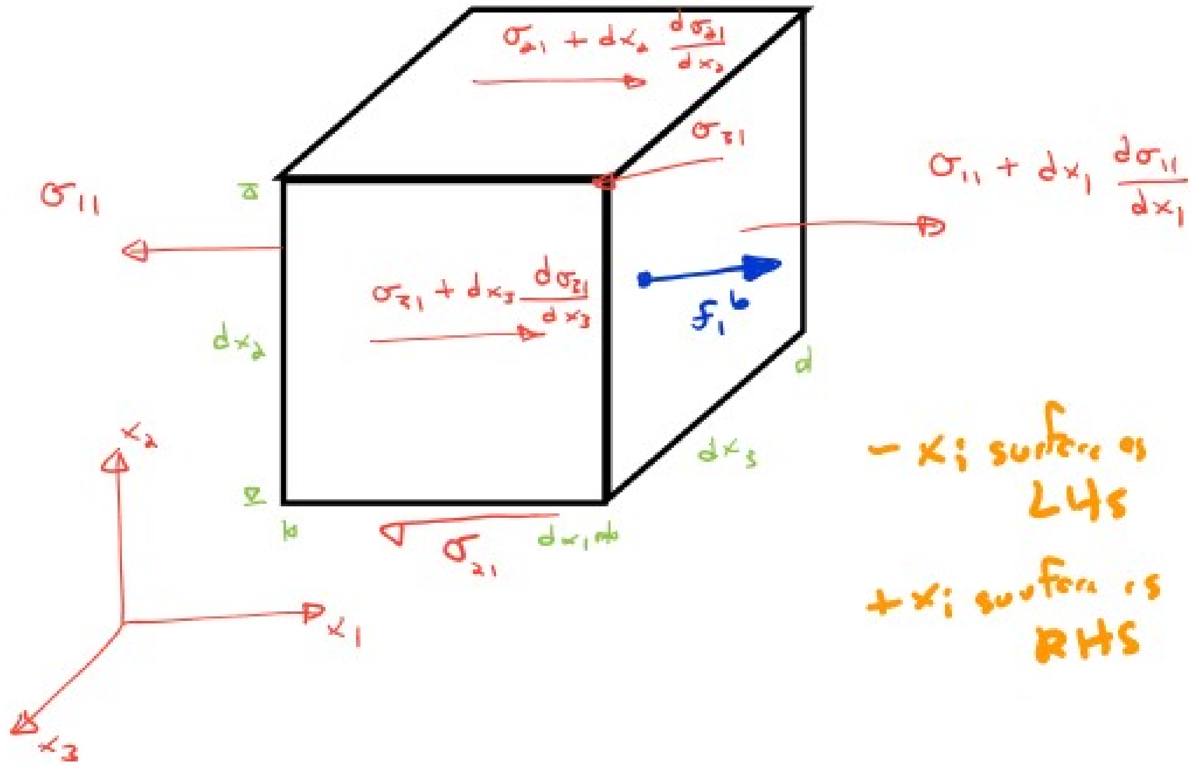
$$\cancel{+ \, dx_1^2 \frac{1}{2!} \frac{d^2\sigma_{11}}{dx_1^2}(x_1 + \alpha dx_1, x_2, x_3)} \underbrace{}_{+\text{ others}}, \quad \text{where } 0 \le \alpha \le 1$$

Given a sufficiently small finite volume (dV) such that dx_1 is very small, the second order partial derivative can be neglected.

Thus the surface Force (dF^e) on the RHS becomes

$$\boxed{dF^{RHS} = \overbrace{\Big[\underbrace{\sigma_{11}}_{\text{LHS}} + \underbrace{dx_1 \frac{d\sigma_{11}}{dx_1}}_{\text{Increment}}\Big]}^{\text{stress}} \cdot \underbrace{dx_2 \, dx_3}_{\text{area}}}$$

Now lets consider a 3D element subject to Normal & Shear stress directed in the x_1 direction.



Surface	Area	Stress	dF
$-x_1$	$dx_2\,dx_3$	σ_{11}	
$-x_2$	$dx_1\,dx_3$	σ_{21}	
$-x_3$	$dx_1\,dx_2$	σ_{31}	
$+x_1$	$dx_2\,dx_3$	$\sigma_{11} + dx_1 \frac{d\sigma_{11}}{dx_1}$	
$+x_2$	$dx_1\,dx_3$	$\sigma_{21} + dx_2 \frac{d\sigma_{21}}{dx_2}$	
$+x_3$	$dx_1\,dx_2$	$\sigma_{31} + dx_3 \frac{d\sigma_{31}}{dx_3}$	

Equilibrium

$$\xrightarrow{+} \Sigma F_x: \left(\sigma_{11} + dx_1 \frac{d\sigma_{11}}{dx_1}\right) dx_2\,dx_3 - \sigma_{11}\,dx_2\,dx_3$$

$$+ \left(\sigma_{21} + dx_2 \frac{d\sigma_{21}}{dx_2}\right) dx_1\,dx_3 - \sigma_{21}\,dx_1\,dx_3$$

$$+ \left(\sigma_{31} + dx_3 \frac{d\sigma_{31}}{dx_3}\right) dx_1\,dx_2 - \sigma_{31}\,dx_1\,dx_2$$

$$+ f_1^b\,dx_1\,dx_2\,dx_3 = 0$$

Cancel like terms

$$\xrightarrow{+} \Sigma F_x : \left(\cancel{\sigma_{11}} + dx_1 \frac{d\sigma_{11}}{dx_1} \right) dx_2\, dx_3 - \cancel{\sigma_{11}}\, dx_2\, dx_3$$
$$+ \left(\cancel{\sigma_{21}} + dx_2 \frac{d\sigma_{21}}{dx_2} \right) dx_1\, dx_3 - \cancel{\sigma_{21}}\, dx_1\, dx_3$$
$$+ \left(\cancel{\sigma_{31}} + dx_3 \frac{d\sigma_{31}}{dx_3} \right) dx_1\, dx_2 - \cancel{\sigma_{31}}\, dx_1\, dx_2$$
$$+ f_1^b\, dx_1\, dx_2\, dx_3 = 0$$

$$\xrightarrow{+} \Sigma F_x : \frac{d\sigma_{11}}{dx_1} \underbrace{dx_1\, dx_2\, dx_3} + \frac{d\sigma_{21}}{dx_2} \underbrace{dx_1\, dx_2\, dx_3}$$
$$+ \frac{d\sigma_{31}}{dx_3} \underbrace{dx_1\, dx_2\, dx_3} + f_1^b \underbrace{dx_1\, dx_2\, dx_3}_{dV} = 0$$

Divide by $dV = dx_1\, dx_2\, dx_3$

Balance of Linear Momentum in the x direction

$$\boxed{\frac{d\sigma_{11}}{dx_1} + \frac{d\sigma_{21}}{dx_2} + \frac{d\sigma_{31}}{dx_3} + f_1^b = 0}$$