Lecture 3

Example 1

$$\underline{\underline{S}} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

a) Find the $\sigma_1 > \sigma_2 > \sigma_3$

a1) using $\det(\underline{\underline{S}} - \sigma_n \underline{\underline{I}}) = 0$

a2) using I_1, I_2, I_3

b) Find the $\underline{n}_1, \underline{n}_2, \underline{n}_3$

a)

a1)

$$\det(\underline{\underline{S}} - \sigma_n \underline{\underline{I}}) = 0$$

$$\begin{vmatrix} (1-\sigma_n) & 2 & 3 \\ 2 & (2-\sigma_n) & 0 \\ 3 & 0 & (2-\sigma_n) \end{vmatrix} = 0$$

$$= (1-\sigma_n)\left[(2-\sigma_n)(2-\sigma_n) - \cancel{(0)}(0)\right]$$

$$- (2)\left[2(2-\sigma_n) - \cancel{(3)(0)}\right]$$

$$+ (3)\left[\cancel{2(0)} - 3(2-\sigma_n)\right] = 0$$

Cancel out, collect like terms, to find

$$\sigma_n^3 - 5\sigma_n^2 - 5\sigma_n + 22 = 0,$$

solve for $\sigma_1 > \sigma_2 > \sigma_3$

$$\sigma_1 = 5.14, \quad \sigma_2 = 2.00, \quad \sigma_3 = -2.14$$

a2) I_1, I_2, I_3

$$I_1 = \mathrm{tr}\,\underline{\underline{S}} = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = 5$$

$$I_2 = -\sigma_{xy}^2 - \sigma_{yz}^2 - \sigma_{xz}^2 + \sigma_{xx}\sigma_{yy}$$
$$+ \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx} = -5$$

$$I_3 = \det(\underline{\underline{S}}) = \sigma_{xx}\sigma_{yy}\sigma_{zz} - \sigma_{xx}\sigma_{yz}^2$$
$$-\sigma_{yy}\sigma_{xz}^2 - \sigma_{zz}\sigma_{xy}^2 + 2\sigma_{xy}\sigma_{yz}\sigma_{xz}$$

$$I_3 = -22$$

$$\sigma_n^3 - I_1\sigma_n^2 + I_2\sigma_n - I_3 = 0$$

$$\sigma_n^3 - 5\sigma_n^2 - 5\sigma_n + 22 = 0$$

b) $\underline{n}'$

$$(\underline{\underline{S}} - \sigma_n \underline{\underline{I}})\cdot \underline{n} = 0$$
$$(\underline{\underline{S}} - \sigma_1 \underline{\underline{I}})\cdot \underline{n}' = 0 \quad \leftarrow 3 \text{ Eqn}$$

$$\sqrt{n_x^2 + n_y^2 + n_z^2} = 1 \quad \leftarrow 1 \text{ Eqn}$$

Plug everything in σ_1

$$\begin{bmatrix} (1-\sigma_1) & 2 & 3 \\ 2 & (2-\sigma_1) & 0 \\ 3 & 0 & (2-\sigma_1) \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

① $(1-5.14)n_x + 2n_y + 3n_z = 0$

② $2n_x + (2-5.14)n_y = 0$

③ $3n_x + (2-5.14)n_z = 0$

④ $\sqrt{n_x^2 + n_y^2 + n_z^2} = 1$

in terms of a single n_i

$$\underline{n}' = \begin{bmatrix} 0.657 \\ 0.418 \\ 0.627 \end{bmatrix}$$

Example 2

Find the Octahedral Normal & Shear Stresses of

$$\sigma = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

a) Direct

$$\sigma_{oct} = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} = \boxed{3}$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6\sigma_{xy}^2 + 6\sigma_{yz}^2 + 6\sigma_{xz}^2}$$

$$\boxed{\tau_{oct} = 4.69}$$

b) Using Principal Stress Form

$$|\underline{\underline{\sigma}} - \sigma_n I| = 0 \qquad \sigma_1 = 9.623, \ \sigma_2 = 0, \ \sigma_3 = -0.623$$

$$\sigma_{oct} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \boxed{3}$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$

$$\boxed{\tau_{oct} = 4.69}$$

Example 3

Decompose $\underline{\underline{\sigma}}$ into hydrostatic & deviatoric portions

$$\sigma = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

$$\sigma_H = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \boxed{= 3}$$

$$\underline{\underline{\sigma}}_D = \underline{\underline{\sigma}} - \sigma_H \cdot \underline{\underline{I}}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\boxed{\underline{\underline{\sigma}}_D = \begin{bmatrix} -2 & 2 & 3 \\ 2 & 0 & 4 \\ 3 & 4 & 2 \end{bmatrix}}$$