MECH 5312 – Solid Mechanics II

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Principal Stress

- The principal directions coincide with Arbitrary plane, **n** through the body where the normal stress, σ_n is maximized and shear stress disappears, $\tau = 0$

$$\begin{bmatrix} (\sigma_{xx} - \sigma_n) & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & (\sigma_{yy} - \sigma_n) & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & (\sigma_{zz} - \sigma_n) \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(\mathbf{S} - \sigma_n \mathbf{I})\mathbf{n} = \mathbf{0}$$

$$\sqrt{n_x^2 + n_y^2 + n_z^2} = 1$$

Stress Invariants

- To solve for the principal stresses, σ_1 , σ_2 , and σ_3 take the determinant of the inner portion and solve

$$\det\left(\mathbf{S} - \sigma_n \mathbf{I}\right) = 0 \qquad\qquad \sigma_1 > \sigma_2 > \sigma_3$$

- or use the Stress Invariants

$$\sigma_n^3 - I_1 \sigma_n^2 + I_2 \sigma_n - I_3 = 0$$

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = \mathrm{tr}\,\mathbf{S}$$

$$I_2 = -\sigma_{xy}^2 - \sigma_{yz}^2 - \sigma_{xz}^2 + \sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx}$$

$$I_3 = \det \mathbf{S}$$

Principal Directions

- The principal directions, $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ can then be obtained by plugging each principal stress into

$$\left(\mathbf{S}-\sigma_n\mathbf{I}\right)\mathbf{n}=\mathbf{0}$$

$$\sqrt{n_x^2+n_y^2+n_z^2}=1$$

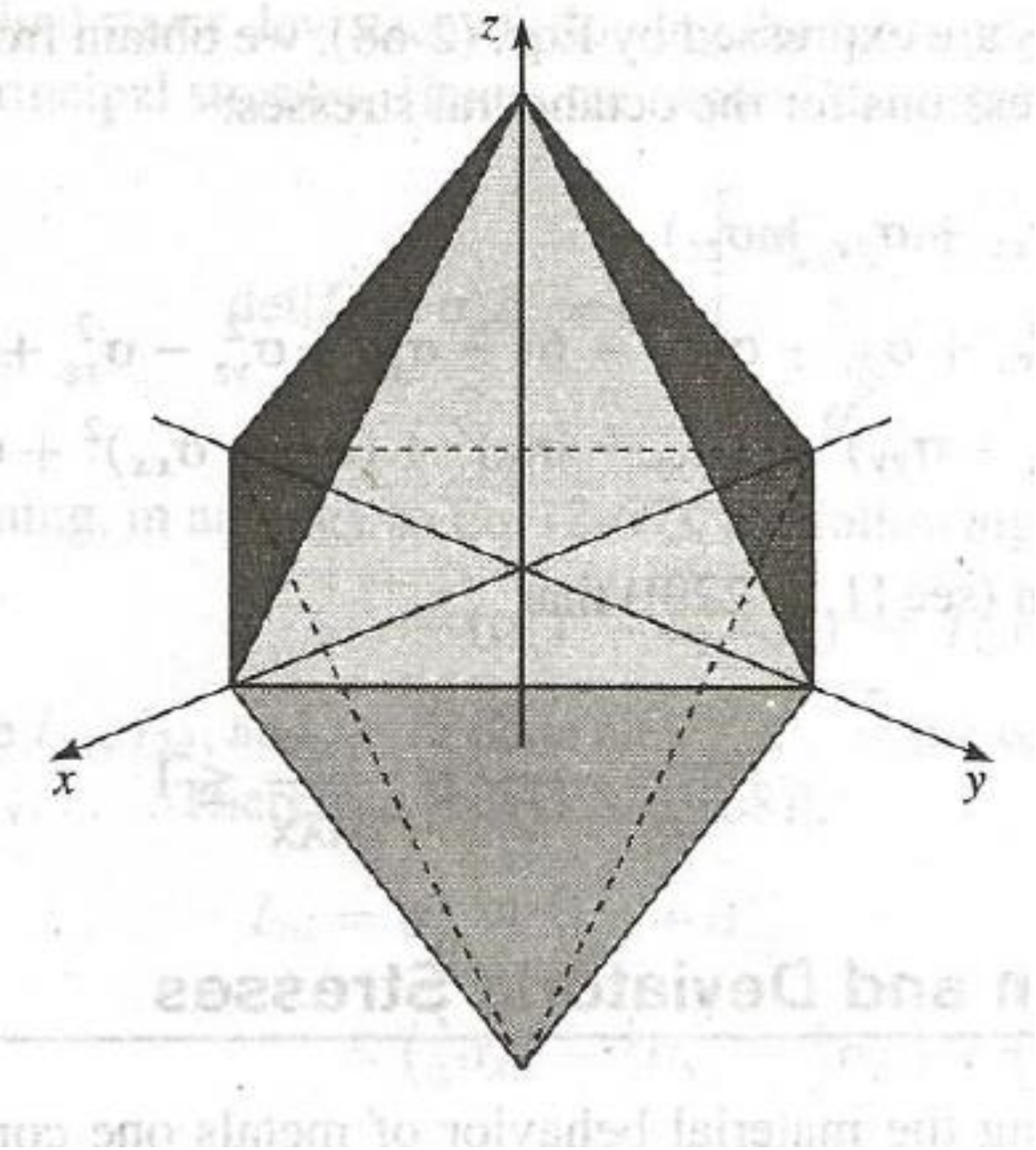
- and solving for **n** using the 4 equations.

Example

2-6 (Solecki)

Octahedral Stresses

- Consider a stress element with only principal stresses.
- If we section this element into a system of planes that are equally inclined with respect to each other.
- Theses are the octahedral planes with the stress element.



Octahedral Stresses

- The Octahedral normal stress and shear stress can be calculated as follows
- In terms of principal stress

$$\sigma_{oct} = \frac{1}{3}I_1 = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

$$\tau_{oct} = \sqrt{\frac{2}{9}I_1^2 - \frac{2}{3}I_2} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$

- In term of general stress

$$\sigma_{oct} = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2)}$$

Example

Hydrostatic Stress

- Cauchy stress can be decomposed into volumetric (hydrostatic/spherical) and shear (deviatoric) parts.

$$\mathbf{S} = \mathbf{S}_d + \sigma_H \mathbf{I}$$

- **Hydostatic stress** is the stress that drives volume change and is often called **mean stress**

$$\sigma_H = \sigma_m = \frac{1}{3} \operatorname{tr} \mathbf{S} = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} = \sigma_{oct}$$

Deviatoric Stress

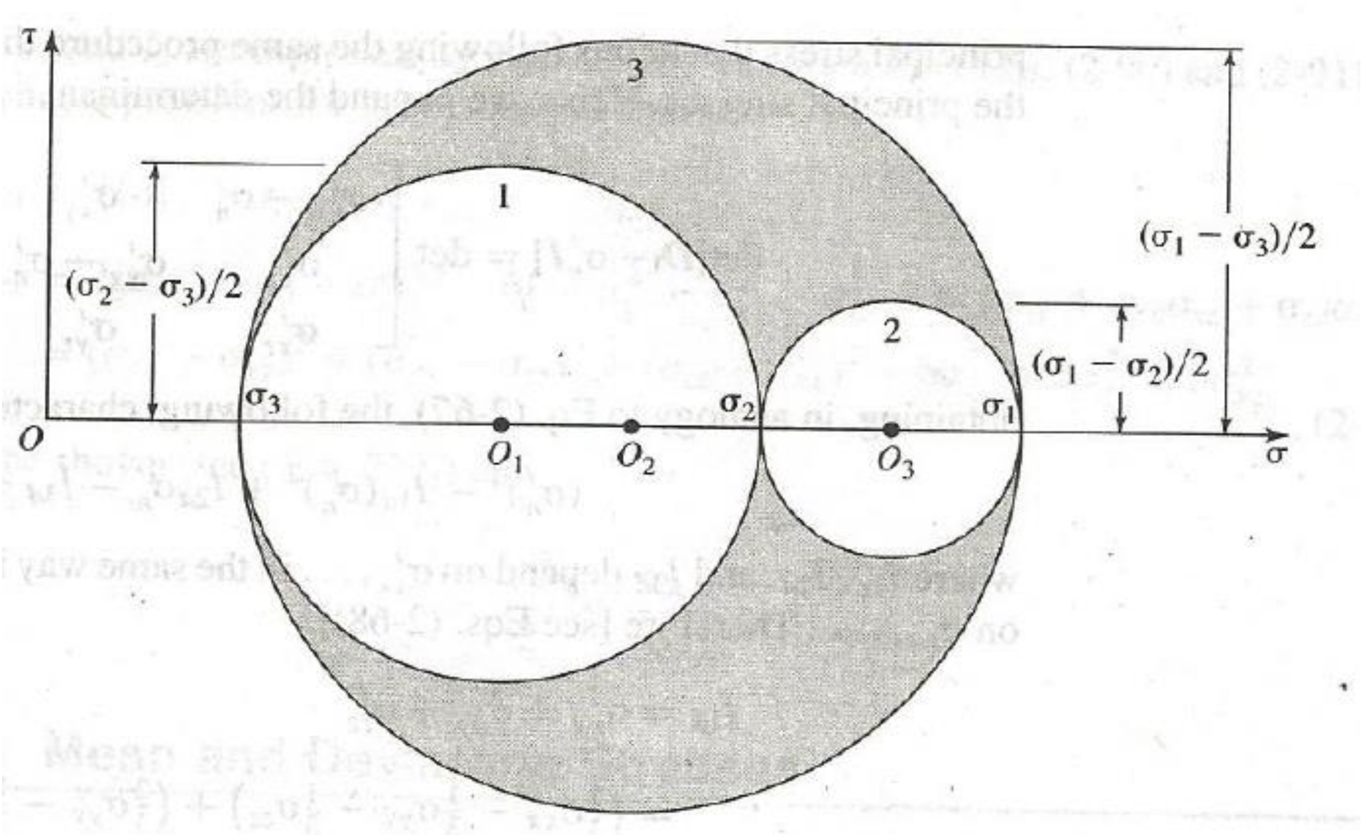
- **Deviatoric stress** is the stress that drives shape change

$$\mathbf{S}_D = \mathbf{S} - \sigma_m \mathbf{I}$$

Example

Mohr’s Circle

- A graphical approach to see how normal and shear stress change with the angle of the arbitrary plane, **n**



$$\tau_{\max,12} = \frac{|\sigma_1 - \sigma_2|}{2}$$

$$\tau_{\max,23} = \frac{|\sigma_2 - \sigma_3|}{2}$$

$$\tau_{abs,\max} = \tau_{\max,13} = \frac{|\sigma_1 - \sigma_3|}{2}$$

Symbolic Manipulation

Open MathCAD

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