Solid Mechanics II

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ORIGIN:= 1

Examples Lecture 5

Example 1

Find the principal strains for the given state of strain using (a) the determinant approach (b) the strain invariant approach

$$\varepsilon := \begin{pmatrix} 1 & 2 & -2 \\ 2 & 3 & 0 \\ -2 & 0 & 4 \end{pmatrix} \cdot 10^{-2}$$

part a)

$$I := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left|\varepsilon - \varepsilon_n \cdot I\right| \rightarrow \frac{2 \cdot \varepsilon_n^{\,2}}{25} - \varepsilon_n^{\,3} - \frac{11 \cdot \varepsilon_n}{10000} - \frac{1}{62500}$$

$$\varepsilon_n := \begin{pmatrix} 1 \\ 0.02 \\ 0 \end{pmatrix}$$

Given

$$\frac{2 \cdot \varepsilon_n^{\,2}}{25} - \varepsilon_n^{\,3} - \frac{11 \cdot \varepsilon_n}{10000} - \frac{1}{62500} = 0$$

$$\text{Find}(\varepsilon_n) = \begin{pmatrix} 0.054 \\ 0.034 \\ -8.595 \times 10^{-3} \end{pmatrix}$$

$$\varepsilon_n := \varepsilon_n$$

part b)

$$J_1 := \varepsilon_{1,1} + \varepsilon_{2,2} + \varepsilon_{3,3} = 0.08$$

$$J_2 := \varepsilon_{1,1}\cdot\varepsilon_{2,2} + \varepsilon_{2,2}\cdot\varepsilon_{3,3} + \varepsilon_{3,3}\cdot\varepsilon_{1,1} - \left[\left(\varepsilon_{1,2}\right)^2 + \left(\varepsilon_{1,3}\right)^2 + \left(\varepsilon_{2,3}\right)^2\right] = 1.1 \times 10^{-3}$$

$$J_3 := \left|\varepsilon\right| = -1.6 \times 10^{-5}$$

$$\varepsilon_n{}^3 - J_1\cdot\varepsilon_n{}^2 + J_2\cdot\varepsilon_n - J_3 \rightarrow 0.0010999999999999999\cdot\varepsilon_n + -0.08\cdot\varepsilon_n{}^2 + \varepsilon_n{}^3 + 0.000016$$

$$\varepsilon_n := \begin{pmatrix} 1 \\ 0.02 \\ 0 \end{pmatrix}$$

Given

$$0.0010999999999999999\cdot\varepsilon_n + -0.08\cdot\varepsilon_n{}^2 + \varepsilon_n{}^3 + 0.000016 = 0$$

$$\text{Find}\left(\varepsilon_n\right) = \begin{pmatrix} 0.054 \\ 0.034 \\ -8.595 \times 10^{-3} \end{pmatrix}$$

The principal strain are the same for both methods.

Example 2

Find the dilatation and deviatoric strain for the following strain tensor.

$$\underset{\sim}{\varepsilon} := \begin{pmatrix} 1 & 2 & -2 \\ 2 & 3 & 0 \\ -2 & 0 & 4 \end{pmatrix} \cdot 10^{-2}$$

part a) Find the Dilatation

$$\Delta := \varepsilon_{1,1} + \varepsilon_{2,2} + \varepsilon_{3,3} = 0.08$$

$$J_1 = 0.08$$

part b) Find the deviatoric strain

$$\varepsilon_d := \varepsilon - \frac{1}{3} \cdot \Delta \cdot I = \begin{pmatrix} -0.017 & 0.02 & -0.02 \\ 0.02 & 3.333 \times 10^{-3} & 0 \\ -0.02 & 0 & 0.013 \end{pmatrix}$$