MECH 5312 – Solid Mechanics II

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Principal Strains

• **Principal stresses** exists on an Arbitrary plane, \( n \) in a body where only normal stresses appear. Find two complementary planes that are orthogonal to \( n \). We arrive at the **principal stress tensor**.

\[
\sigma = \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3 \\
\end{bmatrix}
\]

\( n_1 \quad n_2 \quad n_3 \)

• What form does the corresponding **principal strain tensor** take?
Principal Strain Tensor

- Since, the principal stress tensor contains only normal stresses, the principal strain tensor will contain only normal strains.

\[
\varepsilon = \begin{bmatrix}
\epsilon_1 & 0 & 0 \\
0 & \epsilon_2 & 0 \\
0 & 0 & \epsilon_3
\end{bmatrix}
\]

- Our procedure for finding the principal strains follows
Principal Strain

• The principal directions coincide with Arbitrary plane, \( n \) through the body where the normal strain, \( \varepsilon_n \) is maximized and shear strain disappears, \( \gamma = 0 \)

\[
\begin{bmatrix}
(\varepsilon_{xx} - \varepsilon_n) & \varepsilon_{xy} & \varepsilon_{xz} \\
\varepsilon_{yx} & (\varepsilon_{yy} - \varepsilon_n) & \varepsilon_{yz} \\
\varepsilon_{zx} & \varepsilon_{zy} & (\varepsilon_{zz} - \varepsilon_n)
\end{bmatrix}
\begin{bmatrix}
n_x \\
n_y \\
n_z
\end{bmatrix} = \begin{bmatrix}0 \\
0 \\
0\end{bmatrix}
\]

\((\varepsilon - \varepsilon_n I)n = 0\)

\[
\sqrt{n_x^2 + n_y^2 + n_z^2} = 1
\]
Strain Invariants

• The exact solution to the determinant

\[ \det(\varepsilon - \varepsilon_n \mathbf{I}) = 0 \]

• resolves into a quadratic equation

\[ \varepsilon_n^3 - J_1 \varepsilon_n^2 + J_2 \varepsilon_n - J_3 = 0 \]

• where \( J_1, J_2, \) and \( J_3 \) are the strain Invariants.

\[
\begin{align*}
J_1 &= \text{tr}(\varepsilon) = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} \\
J_2 &= \varepsilon_{xx} \varepsilon_{yy} + \varepsilon_{xy} \varepsilon_{yz} + \varepsilon_{zz} \varepsilon_{xx} - \varepsilon_{xy}^2 - \varepsilon_{xz}^2 - \varepsilon_{yz}^2 \\
J_3 &= \det(\varepsilon)
\end{align*}
\]
Example

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Volume Change and Dilatation

• A differential element is subject to normal strain $\varepsilon_{xx}$ in the x direction
• The length of the side increases by $\varepsilon_{xx} \, dx$
• The new length is now $(1 + \varepsilon_{xx}) \, dx$

Example, for $dx$
Volume Change and Dilatation

- If the differential element is subject to normal strains \( \varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz} \)
- The lengths of the sides increase by \( \varepsilon_{xx} dx, \varepsilon_{yy} dy, \varepsilon_{zz} dz \)
- The new lengths are now \( (1+\varepsilon_{xx})dx, (1+\varepsilon_{yy})dy, (1+\varepsilon_{zz})dz \)

The Initial Volume, \( V \)

\[ V = dx \, dy \, dz \]

The Deformed Volume, \( V_{\text{def}} \)

\[ V_{\text{def}} = (1+\varepsilon_{xx})(1+\varepsilon_{yy})(1+\varepsilon_{zz}) \, dx \, dy \, dz \]
Volume Change and Dilatation

• Assuming finite displacement, the change in volume becomes

\[ \Delta V = V_{\text{def}} - V_0 = (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) \, dx \, dy \, dz \]

Second order terms are neglected!

• The change in volume per unit volume is

\[ \Delta \equiv \frac{\Delta V}{V} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} \]

• where \( \Delta \) is the dilatation and \( \Delta = J_1 \)
Strain Deviator

• Strain can be decomposed into volumetric and deviatoric parts

\[ \epsilon = \left( \frac{1}{3} \Delta \right) I + \epsilon_d \]

• The deviatoric strain can be found by rearranging

\[ \epsilon_d = \epsilon - \left( \frac{1}{3} \Delta \right) I \]
Example

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