MECH 5312 – Solid Mechanics II

Dr. Calvin M. Stewart

Department of Mechanical Engineering

The University of Texas at El Paso

Table of Contents

Principal Strains

- **Principal stresses** exists on an Arbitrary plane, **n** in a body where only normal stresses appear. Find two complementary planes that are orthogonal to **n.** We arrive at the **principal stress tensor.**

$$\begin{array}{c} \quad \mathbf{n}_1 \quad \mathbf{n}_2 \quad \mathbf{n}_3 \\ \boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \begin{array}{c} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \mathbf{n}_3 \end{array} \end{array}$$

- What form does the corresponding **principal strain tensor** take?

Principal Strain Tensor

- Since, the **principal stress tensor** contains **only normal stresses**, the **principal strain tensor** will contain **only normal strains**.

$$\begin{matrix} & \mathbf{n}_1 & \mathbf{n}_2 & \mathbf{n}_3 & \\ \boldsymbol{\varepsilon} = & \left[\begin{matrix} \varepsilon_1 \\ 0 \\ 0 \end{matrix}\right. & \begin{matrix} 0 \\ \varepsilon_2 \\ 0 \end{matrix} & \left.\begin{matrix} 0 \\ 0 \\ \varepsilon_3 \end{matrix}\right] & \begin{matrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \mathbf{n}_3 \end{matrix} \end{matrix}$$

- Our procedure for finding the principal strains follows

Principal Strain

- The principal directions coincide with Arbitrary plane, **n** through the body where the normal strain, ε_n is maximized and shear strain disappears, $\gamma = 0$

$$\begin{bmatrix} (\varepsilon_{xx} - \varepsilon_n) & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & (\varepsilon_{yy} - \varepsilon_n) & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & (\varepsilon_{zz} - \varepsilon_n) \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(\boldsymbol{\varepsilon} - \varepsilon_n \mathbf{I})\mathbf{n} = \mathbf{0}$$

$$\sqrt{n_x^2 + n_y^2 + n_z^2} = 1$$

Strain Invariants

- The exact solution to the determinant

$$\det\left(\boldsymbol{\varepsilon} - \varepsilon_n \mathbf{I}\right) = 0$$

- resolves into a quadratic equation

$$\varepsilon_n^3 - J_1 \varepsilon_n^2 + J_2 \varepsilon_n - J_3 = 0$$

- where J_1, J_2, and J_3 are the strain Invariants.

$$J_1 = tr\left(\boldsymbol{\varepsilon}\right) = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

$$J_2 = \varepsilon_{xx}\varepsilon_{yy} + \varepsilon_{yy}\varepsilon_{zz} + \varepsilon_{zz}\varepsilon_{xx} - \varepsilon_{xy}^2 - \varepsilon_{xz}^2 - \varepsilon_{yz}^2$$

$$J_3 = \det\left(\boldsymbol{\varepsilon}\right)$$

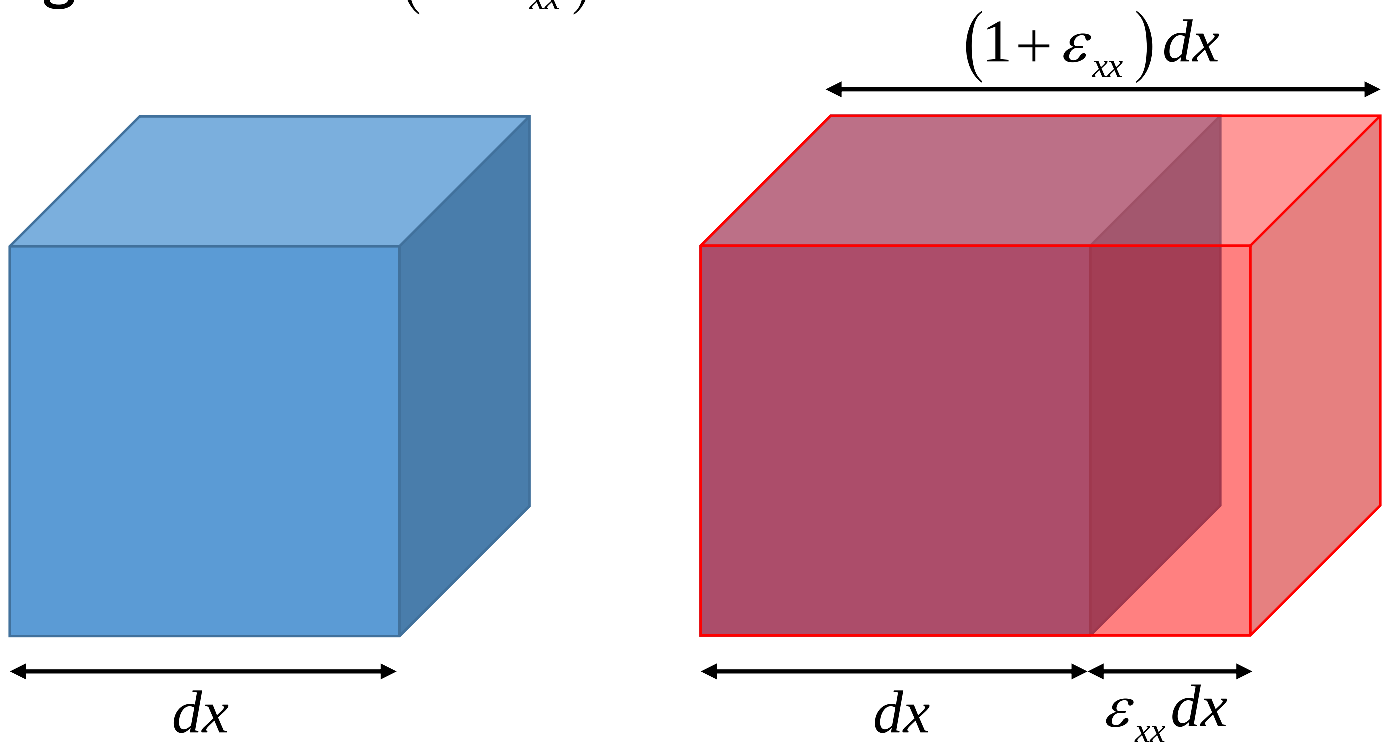
Example

MathCAD

Volume Change and Dilatation

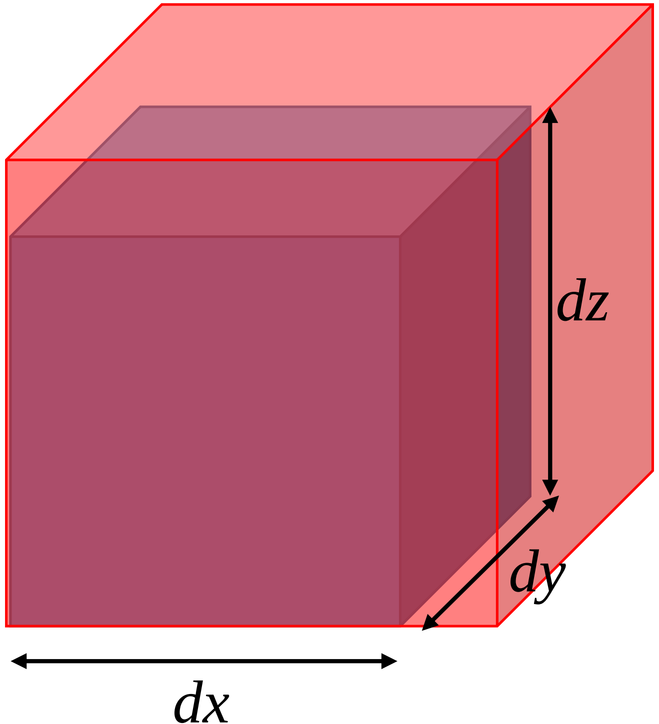
- A differential element is subject to normal strain ε_{xx} in the x direction
- The length of the side increases by $\varepsilon_{xx} dx$
- The new length is now $(1+\varepsilon_{xx})dx$

Example, for dx



Volume Change and Dilatation

- If the differential element is subject to normal strains $\varepsilon_{xx},\ \varepsilon_{yy},\ \varepsilon_{zz}$
- The lengths of the sides increase by $\varepsilon_{xx}dx,\ \varepsilon_{yy}dy,\ \varepsilon_{zz}dz$
- The new lengths are now $(1+\varepsilon_{xx})dx,\ (1+\varepsilon_{yy})dy,\ (1+\varepsilon_{zz})dz$



The Initial Volume, *V*

$$V = dxdydz$$

The Deformed Volume, V_{def}

$$V_{def} = (1+\varepsilon_{xx})(1+\varepsilon_{yy})(1+\varepsilon_{zz})dxdydz$$

Volume Change and Dilatation

- Assuming finite displacement, the change in volume becomes

$$\Delta V = V_{def} - V_0 = \left(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}\right) dxdydz$$

Second order terms are neglected!

$$V = dxdydz$$

- The change in volume per unit volume is

$$\Delta \equiv \frac{\Delta V}{V} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

- where Δ is the dilatation and $\Delta = J_1$

Strain Deviator

- Strain can be decomposed into volumetric and deviatoric parts

$$\boldsymbol{\varepsilon} = \left(\frac{1}{3}\Delta\right)\mathbf{I} + \boldsymbol{\varepsilon}_d$$

- The deviatoric strain can be found by rearranging

$$\boxed{\boldsymbol{\varepsilon}_d = \boldsymbol{\varepsilon} - \left(\frac{1}{3}\Delta\right)\mathbf{I}}$$

Example

MathCAD

Contact Information

Calvin M. Stewart
Assistant Professor
Department of Mechanical Engineering
The University of Texas at El Paso
500 W. University Blvd.
Suite A126
El Paso, Texas 79968-0717

Email: cmstewart@utep.edu
URL: http://me.utep.edu/cmstewart/
Phone: 915-747-6179
Fax: 915-747-5019