MECH 5390 – Fatigue Analysis for Extreme Environments

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Linear Elastic Fracture Mechanics (LEFM) and Fatigue Crack Growth (FCG)

• The Basics of LEFM
• Crack Tip Plasticity
• FCG Testing
• Mechanics of FCG
• Modeling FCG
• Mean Stress
• Crack Closure
• FCG Properties
• Pros and Cons of LEFM Methods
• Beyond LEFM
Basics of Linear Elastic Fracture Mechanics (LEFM)
Basics of LEFM

• The size of the crack at the transition from initiation to propagation is usually unknown and often depends on the point of view of the analyst and the size of the component being analyzed.

• For example, for a researcher equipped with a microscopic it may be the order of a crystal imperfection, dislocation, or a 0.1 mm-crack, while to the inspector in the field it may be the smallest crack that is readily detectable with nondestructive inspection equipment.
Basics of LEFM

• Initiation versus Propogation
• Nevertheless, the distinction between the initiation life and propagation life is important
• At low strain amplitudes up to 90% of life may be taken up with initiation
• At high amplitude the majority of the fatigue life may be spent propagating the crack.
Basics of LEFM

• Order of Crack Progression:
  • initiation
  • propagation
  • rupture

• A substantial portion of the fatigue life can be spent in crack propagation

• Testing with smooth specimens is not appropriate for assessing fatigue crack propagation

• Standardized approaches and empirical methods are needed

Softening in the $\sigma_a$-N curve is sometimes used as an indicator of crack initiation. In the figure below $N_i @ 80\%$ Peak Load
Basics of LEFM

\[ N_f = N_i + N_p \]
Basics of LEFM

• Force-controlled and strain-controlled fatigue are relevant to crack initiation and early propagation

• LEFM Methods are appropriate otherwise
Basics of LEFM

\[ N_{\text{total}} = N_{\text{nucleation}} + N_{\text{growth}} \]

- \( S-N \) or \( \varepsilon-N \)

- \( N_{\text{nucleation}} \) may be 0 or almost the entire life
- \( N_{\text{growth}} \) may be very small or almost the entire life
Basics of LEFM

• Damage tolerant and fatigue crack growth analysis design assume a discontinuity, flaw, or crack exists.

• To make life estimations for fatigue crack growth and damage tolerant design, the following information are often needed:
  • The stress intensity factor, $K$.
  • The fracture toughness, $K_c$.
  • The applicable fatigue crack growth rate expression.
  • The initial crack size, $a_i$ ($a_o$).
  • The final or critical crack size, $a_f$ ($a_c$).
Stress Concentration Effect of Flaws and the need for Fracture Mechanics.

- Consider a plate with an elliptical hole.
- Based on Mechanics of Materials approach for stress concentrations:

\[
\sigma_A = K_t \sigma = (1 + 2 \frac{a}{b}) \sigma
\]

- For circular hole, \( a = b \), \( K_t = 3 \)
- As \( a \gg b \) (i.e. crack), \( \sigma_A \) goes to infinity (even for small \( \sigma \)).
- Therefore a different approach is needed (Fracture Mechanics)
Basics of LEFM

• **Fracture mechanics** is used to evaluate the strength of a structure or component in the presence of a crack or flaw.

• **LEFM** is used for material conditions which are *predominantly linear elastic* during the fatigue process.

• For crack growth or fracture conditions that violate this basic assumption, elastic-plastic fracture mechanics (**EPFM**) approaches are used to describe the fatigue and fracture process.
Fracture Modes

Mode I (Opening)
Mode II (In-Plane Shear)
Mode III (Out-of-Plane Shear)
Mode I ($K_I$)

• Mode I is the crack opening mode
• It is the most common mode, particularly in fatigue, because cracks tend to grow on the plane of maximum tensile stress.
Mode II ($K_{II}$)

- Mode II is the in-plane shearing or sliding mode
Mode III ($K_{III}$)

- Mode III is the tearing or anti-plane shear mode.
- It is associated with a pure shear condition, typical of a round notched bar loaded in torsion.
Mixed Mode

- Combinations of these crack extension modes can also occur.

- An example of mixed mode I-II crack extension is a crack on an inclined plane.
  - If $\beta = 90$, reverts to pure Mode I
Mode I Crack Model

- Stress field on $dx \, dy$ element at crack tip

\[
\begin{align*}
\sigma_x &= \sigma \sqrt{\frac{a}{2r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right) \quad (5-34a) \\
\sigma_y &= \sigma \sqrt{\frac{a}{2r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right) \quad (5-34b) \\
\tau_{xy} &= \sigma \sqrt{\frac{a}{2r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \quad (5-34c) \\
\sigma_z &= \begin{cases} 
0 & \text{(for plane stress)} \\
\nu(\sigma_x + \sigma_y) & \text{(for plane strain)}
\end{cases} \quad (5-34d)
\end{align*}
\]
Stress Intensity Factor

• Common practice to define *stress intensity factor*
  \[ K_I = \sigma \sqrt{\pi a} \]  

• Incorporating \( K_I \), stress field equations are
  \[
  \sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)
  \]  
  \[
  \sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)
  \]  
  \[
  \tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}
  \]  
  \[
  \sigma_z = \begin{cases} 
  0 & \text{(for plane stress)} \\
  \nu(\sigma_x + \sigma_y) & \text{(for plane strain)} 
  \end{cases}
  \]
Fracture Toughness

- **Definition:** The **Stress Intensity Factor (SIF)** is a fracture mechanics parameter used to characterize the strength of the singularity at the crack tip, denoted by $K$.

- **Note:** $K$ depends on each of the following:
  - Sample geometry and size
  - Location and length of the crack
  - Loading mode and distribution

- The subscripts $I$, $II$, or $III$ correspond to the opening, sliding, and tearing modes of fracture, respectively.

- **Definition:** The Critical **Fracture Toughness** is a cracked material’s ability to resist fracture, denoted by $K_c$. It is considered a material property and commonly used in design.

- Units are $ksi\sqrt{in}$ or $MPA\sqrt{m}$

$$ksi\sqrt{in} = 1.099MPA\sqrt{m}$$
Stress Intensity Modification Factor

• Stress intensity factor $K_I$ is a function of geometry, size, and shape of the crack, and type of loading

• For various load and geometric configurations, a stress intensity modification factor $Y$ can be incorporated

$$K_I = Y \sigma \sqrt{\pi a}$$

• In the textbook, the $Y$ factor can be obtained from
  • Dimensionless curves in Fig. 6.3.
  • Mathematical expressions in Table 6.1.
Center-Cracked Plate (CCP) in Tension

- $B$ is the thickness (not shown)
- $Y$ is a dimensionless function of $\alpha = \frac{a}{W}$
- Valid when $\frac{h}{b} \geq 1.5$
- Note:
  
  $\sigma_{nom} = \frac{P}{BW}$

$$Y = \frac{1 - 0.5\alpha + 0.326\alpha^2}{(1 - \alpha)^{\frac{1}{2}}}$$

$$K_I = Y \frac{Pa^{1/2}}{BW}$$
Single Edge Notch Tension (SENT)

- $B$ is the thickness
- $Y$ is a dimensionless function of
  \[ \alpha = \frac{a}{W} \]
- Valid when
  \[ \frac{h}{b} \geq 1 \]
- Note:
  \[ \sigma_{nom} = \frac{P}{BW} \]

\[ K_1 = Y \cdot \frac{Pa^{1/2}}{BW} \]
Double Edge Notch Tension (DENT)

- For the single or double edge crack in a semi-infinite plate ($a/w \rightarrow 0$)

- Where 1.12 is the free edge correction
Compact Tension C(T) Specimen

- $B$ is the thickness
- $Y$ is a dimensionless function of $\alpha = \frac{a}{W}$
- Valid when
  \[
  \frac{a_n}{W} = 0.20 \quad \frac{a}{W} \geq 0.2 \quad \frac{W}{20} \leq B \leq \frac{W}{4}
  \]
- Note:

\[
K_I = \frac{P}{B\sqrt{W}} Y
\]

\[
Y = \frac{(2 + \alpha)}{(1 - \alpha)^{\frac{3}{2}}} \left( 0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 14.72\alpha^4 \right)
\]
TABLE 6.1  \( K \) Expressions for Fig. 6.3

a. Center-cracked plate in tension (for \( 0 < 2a/w < 0.95 \))

Fedderson: \( K_1 = S\sqrt{\pi a} \left[ \sec\left(\frac{\pi a}{w}\right) \right]^{1/2} \)

Irwin: \( K_1 = S\sqrt{\pi a} \left[ \frac{w}{\pi a} \tan\left(\frac{\pi a}{w}\right) \right]^{1/2} \)

where \( S = P/Bw \)

b. Single-edge crack in tension (for \( 0 < a/w < 0.95 \))

\[ K_1 = S\sqrt{a} \left[ 1.99 - 0.41\left(\frac{a}{w}\right) + 18.7\left(\frac{a}{w}\right)^2 - 38.48\left(\frac{a}{w}\right)^3 + 53.85\left(\frac{a}{w}\right)^4 \right] \]

where \( 1.12\sqrt{\pi} = 1.99 \) \( S = P/Bw \)

c. Double-edge crack in tension (for \( 0 < 2a/w < 0.95 \))

\[ K_1 = S\sqrt{a} \left[ 1.98 + 0.36\left(\frac{2a}{w}\right) - 2.12\left(\frac{2a}{w}\right)^2 + 3.42\left(\frac{2a}{w}\right)^3 \right] \]

where \( S = P/Bw \)

d. Single-edge crack in pure bending of a beam (for \( 0 < a/w < 1 \))

\[ K_1 = S\sqrt{a} \left[ 1.99 - 2.47\left(\frac{a}{w}\right) + 12.97\left(\frac{a}{w}\right)^2 - 23.17\left(\frac{a}{w}\right)^3 + 24.8\left(\frac{a}{w}\right)^4 \right] \]

where \( S = Mc/I = 6M/Bw^2 \)
Other Cases

• Geometries
  • Double Edge Notch Tension (DENT)
  • Embedded and Edge Elliptical Crack
  • Etc.

• Load Types
  • Single Edge Notch Bending (SENB)
  • Single Edge Notch Three Point Bending
  • Etc.

• Loading Modes
  • Mode I
  • Mode II
  • Mode III
  • Mixed Mode

• Values of K for various loadings and configurations can be calculated using:

• The theory of elasticity involving
  • Analytical calculations
  • Computational calculations (i.e. FEA)
  • Experimental methods (i.e. photoelasticity)

• Take a Course on Fracture Mechanics
Sample of Geometries

- A variety of thick samples are accepted as test specimens for $K_{ic}$-Testing: (a) compact specimen, (b) disk-shaped compact specimen, (c) single-edge-notched bend SE(B) specimen, (d) Center Crack Tension (CCT) specimen, and (e) arc-shaped specimen.
Fatigue Crack Growth

• Once a crack is present in a material, it will tend to grow under the influence of cyclic loading.

• The crack may be initiated by fatigue, or may be pre-existing from manufacture, or may be caused by an impact, or similar event (e.g., a thermal shock)

• The crack will grow to a critical length then fracture of the component will occur.
Crack Tip Plasticity
Crack Tip Plasticity

\[ K_{\text{max}} \]

\[ K_{\text{min}} \]

TIME

Elastic Singularity Zone

Plastic Wake
Crack Tip Plasticity

• Whether fracture occurs in a ductile or brittle manner, or a fatigue crack grows under cyclic loading, the local plasticity at the crack tip controls both fracture and crack growth.

• In LEFM, the material is assumed to be isotropic and elastic

• The size of the plastic zone must be sufficiently small as to not impact the remote load
Crack Tip Plasticity

• The plastic zone size, $r_y$, can be determined by using the stress field equation for $\sigma_y$, and substituting the yield strength, $S_y$, for $\sigma_y$, and $r_y$ for $r$.

\[
\sigma_y = \frac{K}{\sqrt{2\pi r}}
\]

\[
S_y = \frac{K}{\sqrt{2\pi r_y}}
\]

\[
r_y = \frac{1}{2\pi} \left( \frac{K}{S_y} \right)^2
\]

\[
\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ 1 + \sin \frac{\theta}{2} \sin 3 \frac{\theta}{2} \right]
\]
Crack Tip Plasticity

- Plastic relaxation and redistribution of the stress field occurs in the plastic zone.
  - The stress distribution for $\sigma_y$ shown must shift to the right to accommodate the plastic deformation and satisfy equilibrium conditions.
  - As a result, the actual plastic zone size is approximately \textit{twice} the calculated value.
Crack Tip Plasticity

- Using the stress field equations in Fig. 6.2 and the von Mises or maximum shear stress yield criteria, **plastic zone shape** can be determined.

- The resultant plastic zone shape for mode I using the von Mises criterion is shown.

- For plane stress conditions (where $\sigma_z = 0$) a much larger plastic zone exists compared to plane strain condition, where the tensile stress component, $\sigma_z$, restricts plastic flow.

Figure 6.5 Plastic zone size at the tip of a through-thickness crack.
• Under **monotonic loading**, the plastic zone size, $2r_y$, at the crack tip, in the plane of the crack, is:

$$2r_y = 2 \left[ \frac{1}{2\pi} \left( \frac{K}{S_y} \right)^2 \right] = \frac{1}{\pi} \left( \frac{K}{S_y} \right)^2$$  

For plane stress

$$2r_y \approx \frac{1}{3\pi} \left( \frac{K}{S_y} \right)^2$$  

For plane strain

• The criteria for deciding between plane stress and plane strain are discussed in Section 6.3.

• The value $r_y$ is often called the plastic zone radius.
Crack Tip Plasticity

• An important restriction to the use of LEFM is that plastic zone size at the crack tip must be small relative to the crack length as well as the geometrical dimensions of the specimen or part.

A definite limiting condition for LEFM is that net (nominal) stresses in the crack plane must be less than 0.8Sy (80% of yield strength).

Under monotonic loading \( r_y \leq (1/8)a \).

Under cyclic loading it becomes

\[
ASTM\; Criterion
\]

\[
r_y \leq (W - a) \geq \frac{4}{\pi} \left( \frac{K_{\text{max}}}{S_y} \right)^2
\]

Other restrictions include \( r_y \leq 1/8 \) of \( t \) and \((w-a)\) where \( t \) is the thickness and \((w-a)\) is the uncracked ligament along the plane of the crack.

• Otherwise, a plasticity correction is required for the stress intensity factor, \( K \), or elastic-plastic fracture mechanics may be needed (Section 6.9).
Fatigue Crack Growth (FCG) Testing
Fatigue Crack Growth (FCG) Testing

- **ASTM Standard E399** contains a detailed description of the specimen geometry, experimental procedure, and data collection and techniques used to determine valid $K_{lc}$ values.
- When a specimen or component has a thickness less than that required for plane strain conditions, it will experience either mixed-mode or plane stress conditions depending on thickness.
- Under plane strain conditions, once a critical stress is reached, unstable crack growth occurs.
- Under plane stress conditions where the plastic zone size is greater, the crack may first extend by slow stable crack growth prior to unstable fracture.

- **ASTM Standard E561** provides a recommended practice for plane stress fracture toughness testing.
Fatigue Crack Growth (FCG) Testing
Fatigue Crack Growth (FCG) Testing

FCG Testing

**Def’s:**

- SIF range
  \[ \Delta K = K_{\text{max}} - K_{\text{min}} \]

- SIF amplitude
  \[ K_a = \frac{K_{\text{max}} - K_{\text{min}}}{2} \]

- Mean SIF
  \[ K_m = \frac{K_{\text{max}} + K_{\text{min}}}{2} \]

- SIF ratio
  \[ R_K = \frac{K_{\text{min}}}{K_{\text{max}}} \]

- SIF Amplitude ratio
  \[ A_K = \frac{K_a}{K_m} \]
FCG Testing

• Load Controlled FCG
  • Control stress versus time/cycle
  • Crack length measured using Crack Mouth/Tip Opening Displacement (CMOD/CTOD) Output is crack length

• Remember,

\[ \Delta K_I = f(a) \Delta \sigma \]

• Example, for C(t) Specimen

\[
\Delta K = \frac{\Delta P}{B \sqrt{W}} \left( \frac{2 + \alpha}{1 - \alpha} \right)^{3/2} \left( 0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4 \right)
\]

\[
\alpha = \frac{a}{W}
\]

\[ a_A < a_B \]
\[ f(a)_A < f(a)_B \]
\[ (\Delta K_I)_A < (\Delta K_I)_B \]
FCG Testing

- From the initial $\Delta K_i$, the $\Delta K$ increases up to rupture
- Plot crack growth history versus stress intensity factor range
- $da/dN$ – crack growth rate.
- $\Delta K$ – stress intensity factor range.
- Referred to as a $\Delta K$-Curve
FCG Testing

- Load-Controlled

- Displacement Controlled

- Variable Loading
Critical values of $K$ refer to the condition when a crack extends in a rapid (unstable) manner.

$$K_c = S_c \sqrt{\pi a_c} f\left(\frac{a_c}{w}\right)$$

$S_c$ is the applied nominal stress at crack instability.

$a_c$ is the crack length at instability.

$K_c$ is called **fracture toughness** and depends on the material, temperature, strain rate, environment, and thickness.

This equation provides a quantitative design parameter to prevent fracture involving **applied stress, material selection, and crack size**.
\[ K_c = S_c \sqrt{\pi a_c} f \left( \frac{a_c}{w} \right) \]
The general relationship between fracture toughness, $K_c$, and thickness is shown.

- Thin parts have a high value of $K_c$ accompanied by appreciable "shear lips" or slant fracture.
- As the thickness is increased, the percentage of "shear lips" or slant fracture decreases, as does $K_c$. This type of fracture appearance is called mixed-mode implying both slant and flat fracture.
- For thick parts, the entire fracture surface is flat and $K_c$ approaches an asymptotic minimum value, called the "plane strain fracture toughness" $K_{ic}$.
- Plastic zone sizes at fracture are much larger in thin parts as compared to thick parts.

Figure 6.7 Effect of specimen thickness on fracture toughness.
FCG Testing

• **Plane strain fracture toughness** $K_{lc}$ is considered a true material property because it is independent of thickness.

• In order for a plane strain fracture toughness value to be considered valid, it is required that:

$$a \text{ and } t \geq 2.5 \left( \frac{K_{lc}}{S_y} \right)^2$$

• Approximate thickness required for steels and aluminums to obtain valid $K_{lc}$ values are given in Table 6.2.

• Low strength, ductile materials are subject to plane strain fracture only if they are very thick.
FCG Testing

**TABLE 6.2 Approximate Thickness Required for Valid \( K_{lc} \) Tests**

<table>
<thead>
<tr>
<th>Steel ( S_y ), MPa (ksi)</th>
<th>Aluminum ( S_y ), MPa (ksi)</th>
<th>Thickness mm (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>690 (100)</td>
<td>275 (40)</td>
<td>&gt;76 (3)</td>
</tr>
<tr>
<td>1030 (150)</td>
<td>345 (50)</td>
<td>76 (3)</td>
</tr>
<tr>
<td>1380 (200)</td>
<td>448 (65)</td>
<td>45 (1 3/4)</td>
</tr>
<tr>
<td>1720 (250)</td>
<td>550 (80)</td>
<td>19 (3/4)</td>
</tr>
<tr>
<td>2070 (300)</td>
<td>620 (90)</td>
<td>6 (1/4)</td>
</tr>
</tbody>
</table>
A higher yield or ultimate strength generally produces a decrease in $K_{IC}$, and thus a greater susceptibility for catastrophic fracture!

**Figure 6.8** Locus of plane strain fracture toughness versus yield strength [21].
FCG Testing

• Typical results for a low alloy nuclear pressure vessel steel.
  - As the temperature decreases, $Kc$ usually decreases, while the yield strength increases.
  - Thus, even though unnotched or uncracked tensile strength increases with decreasing temperature, the flaw or crack resistance can be drastically reduced.

Figure 6.9 Variation of $K_{1c}$ with temperature for low alloy nuclear pressure vessel steel A533B [23] (reprinted with permission of E. T. Wessel).
Figure 6.10  Influence of fracture toughness on allowable stress or crack size.
<table>
<thead>
<tr>
<th>Material</th>
<th>Process Description</th>
<th>( S_y ) (MPa)</th>
<th>( S_y ) (ksi)</th>
<th>( K_{tc} ) (MPa(\sqrt{m}))</th>
<th>( K_{tc} ) (ksi(\sqrt{m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>11V41(h)</td>
<td>666°C temper</td>
<td>670</td>
<td>97</td>
<td>113</td>
<td>(104)</td>
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<tr>
<td>11V41(f)</td>
<td>As-forged</td>
<td>524</td>
<td>76</td>
<td>67</td>
<td>(62)</td>
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<tr>
<td>4340</td>
<td>425°C temper</td>
<td>1360-1455</td>
<td>(197-211)</td>
<td>79-89</td>
<td>(72-81)</td>
</tr>
<tr>
<td>4340</td>
<td>350°C temper</td>
<td>1380</td>
<td>(200)</td>
<td>66-68</td>
<td>(60-62)</td>
</tr>
<tr>
<td>4340</td>
<td>260°C temper</td>
<td>1495-1640</td>
<td>(217-238)</td>
<td>50-63</td>
<td>(45-57)</td>
</tr>
<tr>
<td>4330V</td>
<td>275°C temper</td>
<td>1400</td>
<td>(205)</td>
<td>85-92</td>
<td>(77-84)</td>
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<tr>
<td>300M</td>
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<td>(236)</td>
<td>56-57</td>
<td>(51-52)</td>
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<tr>
<td>300M</td>
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<td>(240)</td>
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<td>10Ni (vhs)</td>
<td>560°C temper</td>
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<td>(257)</td>
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<td>(49-51)</td>
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<td>18Ni (200)</td>
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<td>Ph13-8Mo</td>
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**Aluminum**

<table>
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<tr>
<th>Material</th>
<th>Process Description</th>
<th>( S_y ) (MPa)</th>
<th>( S_y ) (ksi)</th>
<th>( K_{tc} ) (MPa(\sqrt{m}))</th>
<th>( K_{tc} ) (ksi(\sqrt{m}))</th>
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<td>(76-78)</td>
<td>22-27</td>
<td>(20-25)</td>
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<td>(54-56)</td>
<td>31-44</td>
<td>(28-40)</td>
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<td>2024-T851</td>
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<td>455</td>
<td>(66)</td>
<td>22-28</td>
<td>(21-25)</td>
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<td>2134-T851</td>
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<td>440-460</td>
<td>(64-67)</td>
<td>27-36</td>
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<td>7050-T7351</td>
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<td>460-510</td>
<td>(67-74)</td>
<td>33-41</td>
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<td>7075-T651</td>
<td></td>
<td>515-550</td>
<td>(75-81)</td>
<td>27-31</td>
<td>(25-28)</td>
</tr>
<tr>
<td>7075-T7351</td>
<td></td>
<td>400-455</td>
<td>(58-68)</td>
<td>31-35</td>
<td>(28-32)</td>
</tr>
<tr>
<td>7079-T651</td>
<td></td>
<td>525-540</td>
<td>(76-78)</td>
<td>29-33</td>
<td>(26-30)</td>
</tr>
<tr>
<td>7178-T651</td>
<td></td>
<td>560</td>
<td>(81)</td>
<td>26-30</td>
<td>(24-27)</td>
</tr>
<tr>
<td>A356-T6</td>
<td></td>
<td>217-229</td>
<td>(31-33)</td>
<td>17-18</td>
<td>(15.5-16.5)</td>
</tr>
</tbody>
</table>

**Titanium**

<table>
<thead>
<tr>
<th>Material</th>
<th>Process Description</th>
<th>( S_y ) (MPa)</th>
<th>( S_y ) (ksi)</th>
<th>( K_{tc} ) (MPa(\sqrt{m}))</th>
<th>( K_{tc} ) (ksi(\sqrt{m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-6Al-4V</td>
<td>Mill annealed</td>
<td>875</td>
<td>(127)</td>
<td>123</td>
<td>(112)</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>Recrystallized</td>
<td>815-835</td>
<td>(118-121)</td>
<td>85-107</td>
<td>(77-97)</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>Annealed</td>
<td>1165</td>
<td>(169)</td>
<td>41-51</td>
<td>(37-47)</td>
</tr>
<tr>
<td>Ti-6Al-6V-2Sn</td>
<td>Mill annealed</td>
<td>990-1070</td>
<td>(144-155)</td>
<td>48-67</td>
<td>(44-61)</td>
</tr>
</tbody>
</table>

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1. *Damage Tolerance Design Handbook, CINDAS/Purdue University, Lafayette, IN, 1994.*
FCG Testing

- In design situations, the stress state may be plane stress, where $K_c$ for the particular thickness is required but is often not available.
  - $K_{lc}$ is often used over $K_c$ because of availability as well as $K_{lc}$ being a more conservative value.
  - However, use of $K_{lc}$ rather than $K_c$ may be inefficient and costly in some situations.
  - Use of $K_{lc}$ or $K_c$ is dependent on the application and the safety critical aspects of the component or structure.
Mechanics of FCG
FCG Testing

- From the initial $\Delta K_I$, the $\Delta K$ increases up to rupture
- Plot crack growth history versus stress intensity factor range
- $da/dN$ – crack growth rate.
- $\Delta K$ – stress intensity factor range.
- Referred to as a $\Delta K$-Curve
Stages of Fatigue Fracture
Stage 1: Initiation

- **Def:** Crack initiation threshold, $\Delta K_{th}$ - value of $\Delta K$ below which Stage 1 behavior will not occur, denoted by $\Delta K_{th}$
- **No crack propagation below $\Delta K_{th}$**
- **Above $\Delta K_{th}$, finite crack propagation**
- Microstructural factors affecting Stage 1:
  - Defects
  - Surface Treatment
  - Dislocation slip characteristics
  - Mean stress (or R)
  - Environment:
    - Liquid metal embrittlement (LME)
    - Hydrogen embrittlement (HE)
Stage 2: Stable Fatigue Crack Growth

- Striations occur by development of slip bands in each cycle, followed by tip blunting, followed by closure.
- Striations are synonymous with beach marks, clamshell marks, and so on.
- **Note:**
- Microstructure and mean stress have less influence on fatigue crack growth behavior in stage 2 than in stage 1.
Stage 2: Striations
Stage 3: Final Fracture

- Acceleration of growth rate with $\Delta K$, approaching catastrophic fracture. Primarily controlled by the fracture toughness, $K_c$ or $K_{ic}$

- Similar to failure under static mode
  - cleavage
  - microvoid coalescence

- Influencing factors:
  - Microstructure
  - Mean Stress
  - Thickness

- Less importance
  - Environment
Modeling FCG
Stages of Fatigue Crack Growth
The Paris Law

- Paul C. Paris (1961) suggested that the increment of crack growth was related to the stress intensity factor; proposed methods for predicting the growth of Stage 2 fatigue cracks.

**Note:**
- Steels: \( m \approx 3 \)
- Al alloys: \( m \approx 4 \)
- Ceramics: \( \Delta K_{th} \approx K_{lc} \)

- The threshold represents an endurance limit

\[
\frac{da}{dN} = C (\Delta K)^m
\]

- Rearrange and Integrate to Solve for Cycles (from initiation to failure)

\[
N_{if} = \int_{a_o}^{a_c} \frac{da}{C (\Delta K)^m} = \int_{a_o}^{a_c} \frac{da}{C \left[ f \left( \frac{a}{W} \right) \Delta \sigma \right]^m}
\]

- To solve must find \( a_c \)

\[
\Delta K = f \left( \frac{a}{W} \right) \Delta \sigma \quad \Rightarrow \quad K_c = f \left( \frac{a_c}{W} \right) \sigma_{\text{max}}
\]
Constant Amplitude Crack Growth

- Obtain appropriate crack growth rate data for material, environment and stress ratio
- Determine starting crack size, $a_o$
- Determine critical crack size, $a_c$
- Determine $\Delta K$ for starting crack size, $a_o$
- Note:
  - If $\Delta K < \Delta K_{th}$ then crack will not grow
  - If $\Delta K > \Delta K_{th}$ for crack growth
- Apply appropriate $\Delta K(t)$ [i.e., $\Delta \sigma(t)$], then integrate to get desired crack growth history
Mean Stress Effects
Walker and Forman Equations

• **Walker**
  - $C$ and $m$ where obtained from $R=0$
  - $m$ generally does not change with $R$
  - $C_0$ and $\gamma$ are material constants

\[
\frac{da}{dN} = C (\Delta K)^m
\]
\[
C = \frac{C_0}{(1 - R)^{m(1-\gamma)}}
\]

• **Forman**
  - Assumes $R > 0$
  - $C_2$ and $m_2$ are material constants
  - Models stages 2 and 3 well

\[
\frac{da}{dN} = C (\Delta K)^{m_2}
\]
\[
C = \frac{C_2}{(1 - R)(K_c - K_{\text{max}})}
\]
Crack Closure
Crack Closure

• When $R < 0$, the crack is closed for a portion of the cycle
• Crack propagation is cannot occur under compressive loads
• Define
  \[ \Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{open}} \]
• Recall
  \[ \Delta K = K_{\text{max}} - K_{\text{min}} \]
• Therefore
  \[ \Delta K \geq \Delta K_{\text{eff}} \]
Crack Closure

- In the early 1970’s Elber observed that the surfaces of fatigue crack close (contact each other) when the remotely applied load is still tensile and do not open again until a sufficiently high tensile load is obtained on the next loading cycle.
Crack Closure

- Note:

\[ \Delta K = f \left( \frac{a}{W} \right) \Delta \sigma \sqrt{\pi a} \]

\[ \Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}} \quad \text{for} \quad R \geq 0 \]

\[ \Delta \sigma = \sigma_{\text{max}} - 0 \quad \text{for} \quad R < 0 \]

- Elber [1971] defined

\[ \Delta K_{\text{eff}} = U \Delta K \]

\[ U = 0.5 + 0.4R \]
Measuring Crack Closure

• Indentations
  • Laser Interferometry
  • local approach, surface-only

• Clip Gage
  • Thickness average measure of closure

• Other Approaches
  • Digital Image Correlation
  • Acoustic Emissions
  • Potential Drop
Fatigue Crack Growth Properties for Several Materials
## Fatigue Properties: Walker

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield $\sigma_0$ (MPa (ksi))</th>
<th>Toughness $K_{ic}$ (MPa$\sqrt{m}$ (ksi$\sqrt{in}$))</th>
<th>Walker Equation $C_0$ mm/cycle (MPa$\sqrt{m})^m$</th>
<th>Walker Equation $C_0$ in/cycle (ksi$\sqrt{in})^m$</th>
<th>$m$</th>
<th>$\gamma$ (R $\geq$ 0)</th>
<th>$\gamma$ (R $&lt; 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Man-Ten steel</td>
<td>363 (52.6)</td>
<td>200 (182)</td>
<td>$3.28 \times 10^{-9}$</td>
<td>$1.74 \times 10^{-10}$</td>
<td>3.13</td>
<td>0.928</td>
<td>0.220</td>
</tr>
<tr>
<td>ROC-100 steel</td>
<td>778 (113)</td>
<td>150 (136)</td>
<td>$8.01 \times 10^{-11}$</td>
<td>$4.71 \times 10^{-12}$</td>
<td>4.24</td>
<td>0.719</td>
<td>0</td>
</tr>
<tr>
<td>AISI 4340 steel ($\sigma_u = 1296$ MPa)</td>
<td>1255 (182)</td>
<td>130 (118)</td>
<td>$5.11 \times 10^{-10}$</td>
<td>$2.73 \times 10^{-11}$</td>
<td>3.24</td>
<td>0.420</td>
<td>0</td>
</tr>
<tr>
<td>17-4 PH steel (H1050, vac. melt)</td>
<td>1059 (154)</td>
<td>120 (109)</td>
<td>$3.29 \times 10^{-8}$</td>
<td>$1.63 \times 10^{-9}$</td>
<td>2.44</td>
<td>0.790</td>
<td>—</td>
</tr>
<tr>
<td>2024-T3 Al$^2$</td>
<td>353 (51.2)</td>
<td>34 (31)</td>
<td>$1.42 \times 10^{-8}$</td>
<td>$7.85 \times 10^{-10}$</td>
<td>3.59</td>
<td>0.680</td>
<td>—</td>
</tr>
<tr>
<td>7075-T6 Al$^2$</td>
<td>523 (75.9)</td>
<td>29 (26)</td>
<td>$2.71 \times 10^{-8}$</td>
<td>$1.51 \times 10^{-9}$</td>
<td>3.70</td>
<td>0.641</td>
<td>0</td>
</tr>
</tbody>
</table>
## Fatigue Properties: Forman

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield $\sigma_0$</th>
<th>Toughness $K_{ic}$</th>
<th>$C_2$ (mm/cycle)</th>
<th>$C_2$ (in/cycle)</th>
<th>$m_2$</th>
<th>$K_c$ (MPa$\sqrt{m}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17-4 PH steel (H1025)</td>
<td>1145 (166)</td>
<td>—</td>
<td>$1.40 \times 10^{-6}$</td>
<td>$6.45 \times 10^{-8}$</td>
<td>2.65</td>
<td>132 (120)</td>
</tr>
<tr>
<td>Inconel 718 (Fe-Ni-base, aged)</td>
<td>1172 (170)</td>
<td>132 (120)</td>
<td>$4.29 \times 10^{-6}$</td>
<td>$2.00 \times 10^{-7}$</td>
<td>2.79</td>
<td>132 (120)</td>
</tr>
<tr>
<td>2024-T3 Al$^1$</td>
<td>353 (51.2)</td>
<td>34 (31)</td>
<td>$2.31 \times 10^{-6}$</td>
<td>$1.14 \times 10^{-7}$</td>
<td>3.38</td>
<td>110 (100)</td>
</tr>
<tr>
<td>7075-T6 Al$^1$</td>
<td>523 (75.9)</td>
<td>29 (26)</td>
<td>$5.29 \times 10^{-6}$</td>
<td>$2.56 \times 10^{-7}$</td>
<td>3.21</td>
<td>78.7 (71.6)</td>
</tr>
</tbody>
</table>
Effect of $T$ and $R$
Grain Size, Frequency, Environment

![Graph showing the relationship between grain size, frequency, and environment](image)

- **AISI 1018 Steel**
- Coarse Grain
- Fine Grain
- $R = 0.1$
- $R = 0.7$

![Graph showing fatigue crack growth rate](image)

**Fatigue crack growth rate**

- Stress-intensity factor range ($\Delta K$), ksi$\sqrt{in}$
- Frequency of loading, Hz
- Tests in air, 0.1 to 10 Hz
FCG Ashby plots
FCG Ashby plots
Design for FCG: Damage Tolerant

• Calculate expected growth rates from $da/dN$ data
• Perform NDE on all critical components
• If crack is found, calculate the expected life of the component
• Replace, rebuild if too close to life limit
Pros and Cons of LEFM Approach

**Advantages of LEFM Approach**
- Only Method that deals directly with the propagation of fatigue cracks
- Estimates can be combined with measurements from NDE to find “Damage Tolerant Design” of a cracked component
- Good insights into fatigue mechanisms

**Disadvantages of LEFM Approach**
- Limited accuracy with crack initiation (apply strain-life or stress-life)
- Estimate of initial crack size has a major influence on the predicted life
- Requires an estimate of SIF which may be difficult to obtain for complicated geometries or materials.
- Method relies on assumptions commensurate with LEFM; method is invalid when plastic (or inelastic) fields near the crack tip are large
Microstructurally Small Cracks
Beyond LEFM
Classification of Fracture Mechanics Regimes

- **LEFM**
  - Linear and time-independent
- **EPFM**
  - Non-linear and time-independent
- **TDFM/NLFM**
  - Non-linear and time-dependent (dynamic, viscoelastic, viscoplastic)
Classes of Fracture Mechanics

• Linear Elastic Fracture Mechanics (LEFM)
  • Assumes that the material is isotropic and linear elastic
  • Stress field near the crack tip is calculated using the theory of elasticity
  • Valid only when the inelastic deformation is “small” compared to the length scale of the crack (i.e., small-scale yielding)

• Key Concepts: Stress Intensity Factor, $K$, Paris Law, etc.
Classes of Fracture Mechanics

• **Elastic-Plastic Fracture Mechanics (EPFM)**
  • Assumes that the material is isotropic and elasticplastic
  • Method is appropriate for structures with relatively large plastic zones
  • Strain energy fields or crack tip opening displacements (CTOD) are used to predict crack behavior

• **Key Concepts: Strain Energy Release Rate, J-integral**
Classes of Fracture Mechanics

- **Non-Linear Fracture Mechanics (NLFM) or Time-Dependent Fracture Mechanics (TDFM)**
  - Assumes that the load-displacement behavior of the material is time-dependent due to dynamic load or due to creep
  - Crack tip stress fields vary with time

- **Key Concept:** $C^*$-integral, $C_t$-parameter
EPFM and NLFM

• Elastic-Plastic (EPFM) [Rice, 1968]

\[ J = \int Wdy - T_i \frac{du_i}{dx} ds \]

• Elastic-Creep (NLFM) [Bassani et al., 1981]

\[ C(t) = \lim_{r \to \infty} \int_{\Gamma} \dot{W}dy - T_i \frac{\dot{u}_i}{dx} ds \]
DOS AND DON'TS IN DESIGN

• Do recognize that the presence of cracks or crack-like manufacturing and metallurgical discontinuities can significantly reduce the strength of a component or structure.

• LEFM can aid both qualitatively and quantitatively in estimating static strength as well as fatigue crack growth life and final fracture.

• Do consider that fracture toughness depends much more on metallurgical discontinuities and impurities than does ultimate or yield strength. Low impurity alloys have better fracture toughness.

• Don't expect doubling thickness or doubling ultimate strength of a component to double the fracture load. Cracks can exist and fracture toughness may drop appreciably with both thickness and ultimate strength increases.

• Do recognize the importance of distinguishing between plane stress and plane strain in fracture mechanics analysis as fracture toughness, crack tip plasticity, and LEFM limitations can be significantly different for the two conditions.
DOS AND DON'TS IN DESIGN

• Don't neglect the importance of nondestructive flaw or crack inspection for both initial and periodic inspection periods.

• Do note that most fatigue crack growth usually occurs in mode I even under mixed-mode conditions, and hence the opening mode stress intensity factor range $\Delta K_I$ is often the predominant controlling factor in FCG.

• Do investigate the possibility of using LEFM principles in fatigue crack growth life predictions even in low strength materials; crack tip plasticity can be small even in low strength materials under fatigue conditions. If plasticity is large, EPFM may be required.

• Do consider the possibility of inspection before fracture. High fracture toughness materials may not provide appreciable increases in fatigue crack growth life, but they do permit longer cracks before fracture, which makes inspection and detection of cracks more reliable.
Assignment

• Problem 6.3 from the text.
• Problem 6.4 from text.
• Problem 6.5 from text.
• Problem 6.9 from the text.
• Problem 6.19 from the text.
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References

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